# ANALYTIC GEOMETRY - VI

## Semester-II

Time Allowed : 3 Hours] [Maximum Marks : 36

**Note :** Attempt **two** questions each from Section A and B carrying 5½ marks each and the entire Section C consisting of 7 short answer type questions carrying 2 marks each.

## Section - A

1. (a) A variable plane passes through a fixed point (p, q, r) and meets the coordinate axes in A, B, C. Show that the locus of the pooint common to the planes through A, B, C and parallel to the coordinate planes is $\frac{p}{x}+\frac{q}{y}+\frac{r}{z}=1$.

   (b) Prove that the plane $14x - 8y + 13 = 0$ bisects the obtuse angle between the planes $3x + 4y - 5z + 1 = 0$ and $5x + 12y - 13z = 0$. (2½, 3)

2. (a) Let A, B, C be the point (3, 2, 1), (–2, 0, – 3) and (0, 0, –2). If volume of the tetrahedron PABC is 5, find the locus of P.

   (b) Find the length and foot of perpendicular from point (7, 14, 5) to the plane $2x + 4y - z = 2$. (2½, 3)

3. (a) Find the equation of the lines passing through (1, –2, 3) and parallel to the planes $x - y + 2z = 5$ and $3x + 2y - z = 6$.

   (b) For what value of $\lambda$ and $\mu$ and the three planes $x + y + z = 6$, $x + 2y + 3z = 10$ and $x + 2y + \lambda z = \mu$

   (i) intersect in a point ? (ii) intersect in a line ?

   (iii) fomr a triangular prism ? (2½, 3)

4. (a) Find angle between the lines

$$\frac{-x+2}{-2}=\frac{y-1}{7}=\frac{z+3}{-3} \text{ and } \frac{x+2}{-1}=\frac{2y-8}{4}=\frac{z-5}{4}.$$

   (b) Show that the shortest didstance between lines

$$\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4} \text{ and } \frac{x-2}{2}=\frac{y-4}{4}=\frac{z-5}{5} \text{ is } \frac{1}{\sqrt{6}}$$ and its equations are

$11x + 2y + 7z + 6 = 0$,

$7x + y - 5x + 7 = 0$. (2½, 3)

## Section - B

5. (a) Find the equation of the sphere which passes through the circle $x^2 + y^2 + z^2 = 5$, $x + 2y + 3z = 3$ and touches the plane $4x + 3y = 15$.

   (b) Find the equation of the sphere which passes through the points (1, –3, 4), (1, –5, 2), (1, –3, 0) and whose centre lies on the plane $x + y + z = 0$. (2½, 3)

6. (a) Show that the spheres $x^2 + y^2 + z^2 = 25$ and

$x^2 + y^2 + z^2 - 24x - 40y - 18z + 225 = 0$ touch each other externally. Also find their point of contact.

(b) Prove that the plane $x + 2y - z = 4$ cuts the sphere $x^2 + y^2 + z^2 - x + z - 2 = 0$ in a circle of radius unity. Also find the equation of the sphere which has this circle as one of the great circle. (2½, 3)

7. (a) Prove that $4x^2 - y^2 + 2z^2 + 3yz + 12x - 11y + 6z + 4 = 0$ represents a cone with vertex $(-1, -2, -3)$.

(b) Find the equation of the right circular cone with vertex $(1, -2, -1)$, semi-vertical angle 60° and the axis

$$\frac{x-1}{3} = \frac{y+2}{-4} = \frac{z+1}{5}.$$ (2½, 3)

8. (a) Find the equation of cone with vertex at the origin and which passes through the curve $x^2 + y^2 + 2z^2 - 1 = 0,\ x - y + 3x = 7$.

(b) A is a point on OX and B on OY so that the angle OAB is constant ($= \alpha$). On AB as diameter a circle is described whose plane is parallel to OZ. Prove that as AB varies the circle generates the cone $2xy - z^2 \sin 2\alpha = 0$. (2½, 3)

**Section - C**

9. Attempt all parts of this section :

(a) Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 9,\ 2x + 3y + 4z = 5$ and the point $(1, 2, 3)$.

(b) Find the equation of the tangent plane at $(-1, 4, -2)$ to the sphere $x^2 + y^2 + z^2 - 2x - 4y + 2z - 3 = 0$.

(c) Prove that a tangent line at any point P of a sphere is perpendicular to the radius through P.

(d) Define cone, and hence right circular cone.

(e) Find the angle between the lines

$$\frac{x}{1} = \frac{-y}{0} = \frac{z}{-1} \text{ and } \frac{x}{3} = \frac{y}{4} = \frac{z}{5}.$$

(f) Prove that the radical planes of four spheres, taken two-by-two, pass through one point.

(g) Write down the equations of the straight line through (1, 2, 3) equally inclined tot he axes. (7×2=14)