ANALYTIC GEOMETRY–VI
Semester – II

Time Allowed : Three Hours] **[Maximum Marks : 40**

Note : The candidates are required to attempt *two* questions each from Section A and B carrying 8 marks each and the entire Section C consisting of 8 short answer type questions carrying 1 makrs each.

Section : A

1. A variable plane is at a constant distance p from the origin and meets the co-ordinate axes at A, B, C. Show that the locus of the centroid of the triangle ABC is $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=\frac{9}{p^2}$. 8

2. A plane drawn through a point P (α, β, γ) and at right angles to OP meets the axes in A, B, C. Prove that the area of the triangle is $\frac{OP^5}{2\pi\beta\gamma}$. 8

3. The line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ intersects the plane $x-y+z=16$ in P and Q is the point $(-1,-2,5)$. Find the distance PQ and the angle which PQ makes with the given line. 8

4. Show that the planes $x+y+z=6, 2x+3y+4z=20, x-y+z=2$ meet in a point. Find the coordinates if that point. 8

Section : B

5. Obtain the equation of the sphere having the circle $x^2+y^2+z^2=9, x+y+z=3$ as a great circle. Also find the its centre and radius. 8

6. Find the equation of the sphere which touches the plane $3x+2t-z+2=0$ at the point P $(1,-2,1)$ and also cuts orthogonally, the sphere $x^2+y^2+z^2-4x+6y+4=0$. 8

7. If α is the semi-vertical angle of the right circular cone which passes through the lines OX, OY, $x=y=z$, then show that $\cos\alpha=\left(9-4\sqrt{3}\right)^{-1/2}$. 8

8. Prove that the equation $ax^2+by^2+cz^2+2ux+2vy+2wz+d=0$ represents a cone if and only if $\frac{u^2}{a}+\frac{v^2}{b}+\frac{w^2}{c}=d$. 8

Section : C

9. Do as directed :

(i) Find the equation of the plane through the points $(-1, 1, 1)$ and $(1, -1, 1)$, perpendicular to the plane $x + 2y + 2z = 5$.

(ii) Find the equation of the plane which is parallel to the x axis and has intercepts 5 and 7 on the y and z–axis, respectively.

(iii) Find the conditions that the line $\frac{x - x_1}{\ell} = \frac{y - y_1}{m} = \frac{z - z_1}{n}$ is parallel to the plane $Ax + By + Cz + D = 0$.

(iv) Find the equation of the plane which is patrallel to the line $\frac{x - 4}{1} = \frac{y + 3}{-4} = \frac{z + 1}{7}$ and passes through the points $(0, 0, 0)$ and $(3, -1, 2)$

(v) Find the equation of two tangent planes to the sphere $x^2 + y^2 + z^2 = 9$ which passes through the line $x + y = 6, x - 2z = 3$.

(vi) Show that the two sphee $x^2 + y^2 + z^2 + 6y + 2z + 8 = 0$ and $x^2 + y^2 + z^2 + 6x + 8y + 4z + 20 = 0$ are orthogonal.

(vii) Show that the equation of the cone with vertex at the origin and base curve $f(x, y) = 0, z = k$ is $f\left(\frac{x}{z}k, \frac{y}{z}k\right) = 0$.

(viii) Find the conditions that the plane $lx + my + nz = 0$ may touch the cone $4x^2 - y^2 + 3z^2 = 0$. 1×8=8