PARTIAL DIFFERENTIAL EQUATIONS - IV
Semester - II

Time Allowed : Three Hours] [Maximum Marks : 40+10=50

Note : Attempt *two* questions each from Section A and B carrying 8 marks each and the entire Section C consisting of 10 short answer type questions carrying 8 marks in all.

Note : Solve the gien differential equations, where the symbols have their usual meaning in Section A, B and C.

Section : A

1. (a) Solve $(p^2 + q^2 + 1) = c^2/z^2$. (b) $x^2(y-z)p + y^2(z-x)q = z^2(x-y)$
2. (a) Solve $z^2 = pqxy + 5$. (b) Solve $z = p^2x + q^2y$.
3. (a) Discuss the solution in series of one standard differential equation.
   (b) Solve $r - 4s + 4t = e^{2x-y}$.
4. (a) Solve $r - t + t\sin^2 x + p\tan x = 0$.
   (b) Solve $q^2 + qs - pt = 0$.

Section : B

5. (a) $2\left[\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y}\right] = \cos(x + 2y) + \cos(2y - x)$.

   (b) Solve $\frac{\partial^2 y}{\partial t^2} - a^2\frac{\partial^2 y}{\partial x^2} = E\sin pt$.

6. (a) Solve $(D^2 - DD' + D' - 1)z = \cos(x + 2y)$.

   (b) Solve $\frac{\partial^3 y}{\partial x^3} - 3\frac{\partial^2 y}{\partial^2 + \partial y} + r\frac{\partial^3 z}{\partial y^3} = e^{x+2y}$.

7. A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form $y = a\sin\left(\frac{\pi x}{l}\right)$ from which it si released at time $t = 0$. Show that the displacement of any point at a distance x from one end at time t is given by
   $(x, t) = a\sin\left(\frac{\pi x}{l}\right)\cos\left(\frac{\pi ct}{l}\right)$.

8. A rod of length l with insulated sides is initially at a uniform temperature 40°. Its ends are suddenly cooled to 0°C and are kept at that temp. Find the temp function u (x, t).

Section : C

9. Do as directed :
   (a) Write down the Lagrange's differential equation.

   (b) Solve $\frac{\partial^2 z}{\partial x^2} = 2xy$.

   (c) Solve $\sqrt{p} + \sqrt{q} = a$, where a is constant.
   (d) Solve $p^2 + q^2 = c^2$, where c is constant.
   (e) Sove $p = e^q$.
   (f) Solve $p + q = \cos x + \cos y$.
   (g) Explain Charpits general methods.
   (h) What do you mean by Homogeneous and Non-homogeneous partial differential equations ?
   (i) Write down two-dimensional Laplace equation.
   (j) Solve $p^2 + q^2 = z$.