LINEAR ALGEBRA - I

(Syllabus - Dec. 2012)

Time : Three Hours] [Maximum Marks : 50

Note : Attempt *five* questions in all selecting *two* questions each from Section A and B carrying 10 marks each, and the compulsory question of Section C consisting of 10 short answer type questions carrying 1 marks each. Use of Scientific Non-programmable calculator is allowed.

Section - A

1. Let R be the field of reals and V be the set of vectors in a plane. Show that V (IR) is a vector space with vector addition as internal binary composition and scalar multiplication of the elements of IR with those of V as external binary composition. 10

2. (a) Find the condition on a, b, c such that the matrix

$$E = \begin{bmatrix} a & b \\ -b & c \end{bmatrix} \text{ is a linear combination of the matrices}$$

$$A = \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}, B = \begin{bmatrix} 1 & 1 \\ -1 & 0 \end{bmatrix} \text{ and } C = \begin{bmatrix} 1 & -1 \\ 0 & 0 \end{bmatrix}.$$ 5

(b) Find the value of k so that the vectors

$$\begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} \text{ and } \begin{bmatrix} k \\ 0 \\ 1 \end{bmatrix} \text{ are L.D.}$$ 5

3. Let w_1 and w_2 be two subspaces of IR^4 where
$w_1 = \{(a, b, c, d) \mid b - 2c + d = 0\}$
$w_2^! = \{(a, b, c, d) \mid a = d, b = 2c\}$
Find a basis and demension of
(i) w_1 (ii) w_2 (iii) $w_1 \cap w_2$. 10

4. State and prove Existence Theorem. 10

Section - B

5. (a) Show that the given mapping is linear transformation
$T : V_3 (IR) \rightarrow V_2 (IR)$ defined by
$T (x, y, z) = (x - y + z, 2x)$. 5

(b) Find T (a, b, c) where $T : IR^3 \rightarrow IR$ is defined by
T (1, 1, 1) = 3
T (1, 1, 0) = – 4),
T (1, 0, 0) = 2. 5

6. State the prove Rank-Nullity Theorem. 10

7. Find all the Eigen values and Eigen vectors of the following matrix:

$$\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}.$$ 10

8. Verify Cayley Hamilton theorem for the matrix

$$A = \begin{bmatrix} 1 & \sqrt{2} & 0 \\ \sqrt{2} & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Use the result to find A^{-1}. 10

Section - C

9. Do as directed :
(a) Write the vector x = (1, 7, –4) as linear combination of vectors $x_1 = (1, -3, 2)$ and $x_2 = (2, -1, 1)$ in the vector space $V_3(R)$.
(b) Define Quotient space.
(c) Let U_1 and U_2 are the subspaces of IR^3, where
$U_1 = \{(x, y, z) \mid x = 0\}$
$U_2^! = \{(x, y, z) \mid x = y = z\}$; x, y, z ∈ IR.
Show that $IR^3 = U_1 \oplus U_2$.
(d) Examine whether the given set of vector in V_3(IR) forms a basis or not :
(1, 0, –1), (1, 2, 1), (0, –3, 2).
(e) What do you mean by Linear transformation ?
(f) Let $T : R^3 \rightarrow R^2$ be defined by
T (x, y, z) = (3x, x – y, 2x + y + z).
Prove that $(T^2 - I)(T - 3I) = 0$.
(g) Define the term Algebraic multiplicity.

(h) Find the minimal polynomial for the matrix

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$

(i) Explain Dual of a vector space.

(j) Are the Eigen values of a Hermitian matrix real ? (10×1=10)