ADVANCED CALCULUS-I

Semester - III

Note: The candidates are required to attempt two questions each from Section A and B carrying 8 marks each and the entire Section C consisting of 8 short answer type questions carrying 1

Time Allowed: Three Hours]

6.

7.

 $2ax + az = 0, x \ge 0.$

Section: A 1. If (a, b) is a point of domain of function f such that: (i) f is continuous at (a, b)
(ii) f exists at (a, b). Then prove that f is differentiable at (a, b). Show that the function: is continuous but not differentiable at (0, 0).

Give an example to show that Young's theorem is sufficient for the equality of f_{xy} and f_{yx} at (a_x) 2. b) but not necessary. Prove that, by the transformations: u = x - ct, v = u + ct, the partial differential equation $\frac{\partial^2 z}{\partial t^2} = C^2 \frac{\partial^2 z}{\partial x^2}$ reduces to $\frac{\partial^2 z}{\partial u \partial v} = 0$, where z is a function of u and v. State Taylor's theorem for functions of two variables. Expand $x^4 + x^2y^2 - y^4$ about the point (1, 1) upto terms of second degree. Find the form of R, the remainder after 2 terms. 8 Show that the minimum and maximum values of function: 4. $f(x, y, t) = (ax + by + cz) e^{-\alpha^2 x^2 - \beta^2 y^2 - \gamma^2 z^2} \text{ are } -\sqrt{\frac{1}{2e} (a^2 \alpha^{-2} + b^2 \beta^2 + c^2 \gamma^{-2})} \text{ and } \sqrt{\frac{1}{2e} (a^2 \alpha^{-2} + b^2 \beta^2 + c^2 \gamma^{-2})}$ 8 (a) Find the value of $\int x^2 y dx + xy^2 dx$ taken in clockwise sense along the hexagon C whose 5. vertices are: $(\pm 3a, 0), (\pm 2a, \pm \sqrt{3}a).$ (b) For a function $f(x, y) = \begin{cases} \frac{1}{2}, & \text{y rational} \\ x, & \text{y irrational} \end{cases}$, $\int_{0}^{1} dx \int_{0}^{1} f \, dy \, does \text{ not exist.}$

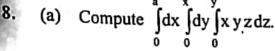
State Green's theorem in plane and hence find the line integral $\int \frac{ydy - ydx}{x^2 + y^2}$ taken in the positive

direction over any closed contour C with origin inside it.

State Stoke's theorem and get Green's theorem as its special case. Use Stoke's theorem to show that:

 $\int (y-z) dy dz + (z-x) dz dx + (x-y) dx dy = \pi a^3$ where S is the portion of the surface $x_2 + y_2 - x$

[Maximum Marks : 35



2

(b) Compute the volume of the ellipsoid
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$
.
Section: C

6

Do as directed: (a) If $x = \gamma \sin \theta \cos \phi$, $y = \gamma \sin \theta \sin \phi$ and $z = \gamma \cos \theta$, find $\frac{\partial (x, y, z)}{\partial (\gamma, \theta, \phi)}$ 9.

(b) Investigate for continuity of the function: $f(x, y) = \begin{cases} x^2 + 2y^2, & (x, y) \neq (1, 2) \\ 0, & (x, y) = (1, 2) \end{cases}$ at (1, 2).

(g) Write the physical interpretation of curl. Write the statement fo Gauss's Divergence theorem.

(e) Define del of a vector valued function.

Show that for differentiable function f, if $z = f(x^2y)$, then (f)

Define homogeneous function of two variables of degree n. Give one example. State necessary and sufficient conditions for a function f of two variables to have extreme value at (a, b). $1 \times 8 = 8$