ANALYSIS–II

Semester - III

Time Allowed : Three Hours] [Maximum Marks : 35

Note : The candidates are required to attempt *two* questions each from Section A and B carrying 8 marks each and the entire Section C consisting of 8 short answer type questions carrying 1 marks.

Section : A

1. (a) Show that a monotonic function in [a, b] is Riemann Intergrable. 4

   (b) Let $f(x) = \begin{cases} 0, \text{ where } x \text{ is rational} \\ 1, \text{ where } x \text{ is irrational} \end{cases}$

   Show that f is not Riemann integrable in any interval. 4

2. (a) If function f is continuous in [0, 1]. Show that : $\lim_{n\to\infty}\int_0^1 \frac{m f(x)}{1+n^2x^2}dx = \frac{\pi}{2} f(0)$.

   (b) State and prove Fundamental Theorem of Integral Calculus. 4

3. (a) Show that the function f (x) defined as below is integrable in [0, 1] :

   $f(x) = \frac{1}{2^n}, \frac{1}{2^{n+1}} < x \le \frac{1}{2^n} (n = 0,1,2....)$

   $= 0 \quad x = 0.$ 4

   (b) Show that sum of two Riemann Integrable functions is Riemann Integrable. 4

4. A function f is Riemann Integrable in [a, – c, a + c] and

   $|f(x)| \le M \, \forall x \in [a-c, a+c]$. Also $\int_{a-c}^{a+b+c} f(x)dx = 0$ and

   $f(x) = \int_{a-c}^{x} f(t)dt$. Prove that $\left[\int_{a-c}^{a+c} f(x)dx\right] \le Mc^2$. 8

Section : B

5. (a) Show that the real sequence $\{x_n\}$ converges if and only $\{x_n\}$ is Cauchy sequence. 4

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(b) Suppose $x_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \dots\dots + \frac{1}{n!}$. Show that $\{x_n\}$ is convergent and $2 \leq \lim x_n \leq 3$. 4

6. Let $\{u_n\}$ be a monotomic sequence and $\lim u_n = 0$ then prove that $\sum_{n=1}^{\infty}(-1)^{n+1} u_n$ is convergent. Hence prove that the series $\sum_{n=1}^{\infty}\frac{(-1)^{n+1}}{n^{\alpha}}$ converges if and ony if $\alpha > 0$. 4

7. Define absolute convergence of a series. Show that if a series converges absolutely then it converges. Is the converse true ? Justify. 8

8. (a) State Able's Test of convergence of series. Show that $\sum_{n=1}^{\infty} n^{1/n}$ a_n is convergent if $\sum_{n>1}^{\infty} a_n$ is convergent. 4

(b) With the help of Dirichlet's test, show that the series $1 - \frac{1}{3.2} + \frac{1}{5.2^2} - \frac{1}{7.2^3} + \dots\dots\dots$ is convergent. 4

Section : C

9. Do as directed :

(a) Show that a constant function is Riemann Integrable.

(b) Let P be a partition of [a, b] and f : [a, b] → R be bounded function . If P* is a refinement of P, show that L (p, f) ≤ L (P*, f).

(c) Show that $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{3} - - - -$ is convergent.

(d) State Gauss Test of convergence of series.

(e) Show that $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2^n}$ is conditionally convergent.

(f) Find the limit of sequence $\left\{\sqrt{n^2+n} - n\right\}$.

(g) Give an example of bounded sequence which is not convergent.

(h) Suppose that f and g are Riemann Integrable on [a, b] and $f \leq g$ on [a, b]. Then $\int_a^b f \leq \int_a^b g$. 1×8=8

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