DIFFERENTIAL EQUATIONS - II

Time : Three Hours] [Maximum Marks : 100

Note : Attempt *one* question each from Sections A, B, C, and D carrying 20 marks each, and the entire Section E consisting of *seven* short answer type questions carrying 20 marks in all.

Section : A

1. (a) Solve in series the differential equation $\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2xy = 0$

   (b) Prove that $F(l, m, p; 1) = \frac{\Gamma(p)\Gamma(p-\ell-m)}{\Gamma(p-\ell)\Gamma(p-m)}$. (10+10)

2. (a) Find the eigenvalues and eigenfunctions of the Strum Liouville problem $\frac{d^2y}{dx^2} + \lambda y = 0,\ y(0) =$

0 and y (α) = 0.

(b) Prove that (2n + 1) $P_n(x) - P'_{n-1}(x) - P'_{n-1}(x)$. (12+8)

Section : B

3. (a) Solve By Charpit's method $(p^2 + q^2)\, y = qz$.
(b) Find he general solution of the partial differential equation.
$\left(D_x^2 - 3D_xD_y = 2D_y^2\right)z = e^{2x-y} + e^{x+y} + \sin(2x+3y)$ (10+10)

4. (a) Find the general solution of the partial differential equation
$x^2r - xys - 2y^2t + xp - 2yq = \log\frac{y}{x} - 4.$
(b) Find the general solution of r + 7s + 12t = 0 (10+10)

Section : C

5. (a) Let f (t) be a piecewise continuous on [0, ∞), be of exponential order and periodic with period T. Then $L[f(t)] = \frac{1}{1-e^{-ST}}\int_0^T e^{-ST} f(t)\,dt, S > 0.$
(b) Using Laplace transformation solve $\frac{d^2y}{dt^2} + t\frac{dy}{dt} - y = 0,$ when y (0) = 0, y′ (0) = 2.

6. (a) Solve the integral equation $f(t) = 1 + t + 2\int_0^t \sin u(t-u)\,du$, using Convolution theorem.
(b) Find Laplace transform of $t\int_0^t e^{-u}\sin 2u\,du$. (10+10)

Section : D

7. (a) Find shortes distance between point P (0, 0) and the straight line $x + y = \sqrt{2}$.
(b) Using Eule's equation find curve passing through (0, 5) and (2, 3) such that its length between these points is shortest. (10+10)

8. (a) Find the shortest distance from the point (2, –8) to the parabola $y^2 = 4x$.
(b) Find the external of $\int_0^1 \left(y'^2 + z'^2 + 2y\right)dx$ with y (0) = 1, y (1) = 3/2, z (0) = 0 and z (1) = 1. (10+10)

Section : E

9. Do as directed :
(a) Show that $J'_1(x) = J_0(x) - \frac{1}{x}J_1(x)$. (3)
(b) Show that $J_0^2 + 2\left(J_1^2 + J_2^2 + \ldots\ldots\ldots\ldots\right) = 1$. (3)
(c) Prove that $\int_{-1}^{1} xP_n(x)P'_n(x)\,dx = \frac{2n}{2n+1}$. (3)
(d) Solve the Lagrange's linear equation px + qz = –y. (2)
(e) Find he Laplace transform of $12t^3 \cosh\frac{t}{2}$. (3)
(f) Find the inverse Laplace transform of $\log\left(1 + \frac{16}{s^2}\right)$. (3)
(g) Find extermals of $\int_a^b \left(xy' + y'^2\right)dx$. (3)