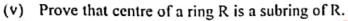
## ALGEBRA

## Paper-I Semester-V

		Semester-v	20
Time	Allov	wed: 3 Hours] [Maximum Mark	S:30
Note	. Th	he candidates are required to attempt two questions each from Sections A and B carryll	ng 5/2
	m	parks each and the entire Section C consisting of 10 short answer type questions carrie	1g 174
		nark each.	
		SECTION-A	
1.	(a)	Prove that set of positive integers which are less than n and co-prime to n forms an a	belian
		crown under energtion of multiplication modillo II.	-
	(b)		e that
	(-)	O(ab) = O(a)O(b).	21/2
		0(H)0(K)	_
2.	(a)	If H and K are finite subgroups of a group G, then $0(HK) = \frac{0(H)0(K)}{0(H0K)}$ .	3
٠.	(4)		21/2
	(þ)	Show that the set of n-nth roots of unity froms an cyclic group under multiplication.	G. 3
3.	(a)	If H is a subgroup of G, then prove that G is equal to union of all right coset of H in G	nfinite
	(b)	Let Q* denotes the set of all rational numbers except-1. Show that Q* forms an i	21/2
		abelian group under the operation * defined by a * b = a + b + ab, for a, b $\in$ Q*.	
4.	(a)	Let G be a finite group and $a \in G$ . Then $O(a)/O(G)$ i.e., the order of an element of a group and $a \in G$ .	3
		a divisor of the order of the group.	N ≃ H
	(b)	If N is a normal subgroup of a group G and H be any subgroup of G <sub>n</sub> . V, then (HN)/N	21/2
		(H ∩ N).	2/1
		SECTION-R	ina of
5.	(a)	If R is a ring in which $x^2 = x$ for all $x \in R$ , prove that R is a commutative r	111g 01
	(-)	abanataristic ?	21/2
	(b)	If I and J are two ideals of a Ring R, then IJ is an ideal of R. Moreover $IJ \subseteq I \cap J$ .	
6.	(a)	the ring / of integers, where a, b are positive in	itegers
٥.	(4)	Determine:	
		(i) 1+J	3
		žis tr	21/3
	(b)	If A, B are any ideals of ring R and B ⊆ A, then A/B is an ideal of R/B.	
	3	in the state of the ring 37. Describe the quotient ring $\frac{32}{127}$	3
7.	(a)	Show that 122 is all ideal of the ring best of all rings is an equivalence relation.	21/
	(b)	Show that relation of Isomorphism in the set of all rings is an equivalence relation.	-
8.	(a)	If I and J be two ideals of ring R, then $I/(I \cap J) \cong I + J/J$ .	_
٠.	()	$\int a^{-1} \int a^{-2} t$	ગે.
	11	Show that the mapping $f: Z   \sqrt{2}  \rightarrow M_2(R)$ defined by $f(a+b\sqrt{2}) = b$	is
	(0)	Show that the mapping $f: Z\left[\sqrt{2}\right] \to M_2(R)$ defined by $f(a+b\sqrt{2}) = \begin{bmatrix} a & 2b \\ b & a \end{bmatrix}$	20
		A homomorphism of imas, i ma item.	
	77	SECTION-C 100	(11/4=14
9.	An	nswer the following questions:	
	(i)	Drove that in a finite ID order of every element exist.	
	(ii)	a Ctate and prove Reversal law of product.	
	(iii	i m i i i i i i i i i i i i i i i i i i	
	(iv		
	-	·	



(vi) Intersection of two left ideal of a ring is left ideal.

(vii) Define Associate and Irreducible elements of a Ring.

(viii) Find units of the Ring Z[i].

(ix) Define Division ring with example.

(x) Show that (Z, +) is a group.