ABSTRACT ALGEBRA - II
(Re-appear April-2013)

Time Allowed : Three Hours **Maximum Marks : 100**

Note : The candidates are required to attempt one question each from Sections A, B, C and D carrying 20 marks each and the entire Section E consisting of 10 short answer type questions carrying 2 marks each.

Section-A

I. (a) State and prove Lagrange's Theorem for groups. 10

(b) Show that the converse of Lagrange's Theorem holds for cyclic groups. 10

II. (a) Let Inn(G) be the group of inner automorphisms of group G, then prove that Inn(G) = G/Z(G), where Z(G is the centre of G. 10

(b) Prove that if order of group G is p^n, p prime, then Z(G) = {e}, where Z(G) is the centre of G 10

Section-B

III. (a) Define an ideal of ring R. Show that for an ideal I of ring R, R/I is a ring and is a homomorphic image of R. 10

(b) Show that in a ring with identity, every proper ideal is contained in a maximal ideal. 10

IV. (a) Prove that in a principle Ideal Domain and element is prime if and only if it is irreducible. 15

(b) Show that the polynomial $x^5 + 2x^3 + 4x^2 + 2$ is irreducible over the field of rationals. 5

Section-C

V. (a) Prove that every subspace of a finite dimensional vector space has a complement. 10

(b) In $V = IR^3$, let $W = \{(\lambda_1, \lambda_2, \lambda_3 \in IR^3 | \lambda_1 = 0)\}$. Determine if W is subspace of V or not. Find a basis of W (if exists). 10

VI. (a) Prove that every subspace of a finite dimensional vector space has a complement. 10

(b) In $V = IR^3$, let $W_1 = \{(x, y, 0) | x, y \in IR)\}$ Show that W_1 is subspace ov V and hence find the complement of W_1. 10

Section-D

VII. (a) Let V be an n-dimensional vector space over field F and W be n-dimensional vector space. Then prove that the space L (V, W) of linear transformations from V into W is vector space of dimension mn. 15

(b) Show that every n-dimensional vector space over field F is siomorphic to F^n. 5

VIII. (a) Let T be the linear operator D^2 defined by $T(x_1, x_2) = (x_1, 0)$. Let β be the standard ordered basis for D2 and $\beta = \{\alpha_1, \alpha_2\}$ be the ordered basis defined by $\alpha_1 = (1, i)$ and $\alpha_2 = (-i, 2)$. Find the matrix of T in the ordered basis β. 10

(b) Let T be the linear operator in IR^3 represented by the matrix $\begin{pmatrix} -9 & 4 & 4 \\ -8 & 3 & 4 \\ -16 & 8 & 7 \end{pmatrix}$ Prove that T is diagonalizable.

Section-E

IX. Do as directed :

(a) Let G be the cyclic group of order 12 generated by a. Find the order of a^2 and a^3.

(b) Show that every subgroup of an abelian group is normal.

(c) Find the Kernel of homomorphism $f : IR \to IR^+$ defined by $f(x) = e^x$.

(d) Does there exist intergral domain with 12 elements ?

(e) Give an example of prime ideal of a ring which is not maximal.

(f) Give an example of :

(i) A ring with untiy (ii) A ring without unity.

(g) Give an example of an Integral Domain which is not Unique Factorization.

(h) Prove tha if two vectors are linearly dependent, then one of them is scalar multiple of another.

(i) Let $T : IR^2 \to IR^2$ be the linear transformation defined by $T(x, y) = (x + y, x - y)$. Find rank (T) and null (T).

(j) Under what conditions on 'a' are the vectors (a, 1, 0), (1, a, 1) and (0, 1, a) in IR^3 are linearly dependent ? 10×2=20