ALGEBRA-I

Semester-V

Time Allowed : 3 Hours] [Maximum Marks : 40

Note : The candidates are required to attempt *five* questions in all, selecting *two* questions each from Section A and B of the questions paper and the entire Section-C. All questions will carry equal marks.

Section - A

1. Let A(S) denotes the set of all permutations on a non-empty set S, then A(S) froms a group under the operation of composition of maps.
2. (a) State and prove Fermat's theorem.
   (b) Let a, b and x be any elements of a group G. Then prove that $O(a) = O(x^{-1}ax)$.
3. (a) The centre Z(G) of a group G is a normal subgroup of G.
   (b) Prove that every subgroup of a cyclic group is cyclic.
4. State and prove Third fundamental theorem on Homomorphism.

Section - B

5. Prove that the set M of all $n \times n$ matrices over reals is a non-commulative ring with unity with zero divisors under addition and multiplication of matrices.
6. (a) Prove that every division ring is simple ring.
   (b) Let $I = \{6n : n \in Z\}$ be an ideal of Z. Write the composition table for the quotient ring Z/I.
7. State and prove second Isomorphism theorem.
8. (a) Find the field of quotient of integral domain $Z[\sqrt{5}]$.

(b) **Prove that every irreducible element in a principal ideal domain is a prime element.**

Section - C

9. (a) Define Permutation group with example. 1

(b) Define order of element of a group with example. 1

(c) State the Einstein criterion. 1

(d) If F is a field, prove that its only ideals are (0) and F itself. 1

(e) Prove that intersection of two left ideals of a group again a left ideal of the group. 1

(f) Prove that every field is an integral domain. 1

(g) State first Isomorphism theorem. ½

(h) Find the units of Z_8. ½

(i) Define prime and irreducible element with example. ½

(j) Define characteristics of a Ring. ½