

**DISCRETE MATHEMATICS- III (i)**  
**(Re-appear April-2013)**

**Time Allowed : Three Hours**

**Maximum Marks :100**

**Note :** The candidates are required to attempt one question each from Sections A, B, C and D carrying 20 marks each and the entire Section E consisting of 10 short answer type questions carrying 2 marks each.

**Section-A**

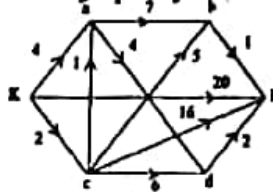
- I. (a) A survey was conducted among 1000 people. Of these 595 like Metro Channel, 595 like Star Movies and 550 like Zee TV, 395 of them like Metro Channel and Star Movies, 350 of them like Metro Channel and Zee TV and 400 of them like Star Movies and Zee TV, 250 of them like Metro Channel, Star Movies and Zee TV. How many of them who do not like Metro Channel, do not like Star Movies and do not like Zee TV? How many of them who like Metro Channel, do not like Star Movies and do not like Zee TV?
- (b) Determine whether the relation  $R = \{(a, b) \in \mathbb{R}, a - b \leq 1\}$  on the set  $I_+$  (set of all positive integers) is reflexive, symmetric, transitive, anti-symmetric, a partial order relation and an equivalence relation. 10+10
- II. (a) In a shipment, there are 40 floppy disks of which 5 are defective. Determine :
- (i) in how many ways we can select five floppy disks?
  - (ii) in how many ways we can select five non-defective floppy disks?
  - (iii) in how many ways we can select five floppy disks containing exactly three defective floppy disks?
  - (iv) in how many ways we can select five floppy disks containing at least 1 defective floppy disk?
- (b) In how many ways 10 programmers can sit on a round table to discuss the project so that project

leader and a particular programmer always sit together.

10+10

- III. (a) State and prove Euler's Formula for planar graph.  
 (b) Find the shortest path between K and J in graph by Dijkstra's Algorithm.

10+10



- IV. (a) An undirected graph possesses an Eulerian circuit if and only if it is connected and its vertices are all of even degree. Prove with the help of a suitable example.

- (b) Let  $G[V, E]$  be a graph having at least 11 vertices. Prove that  $G$  and its complement  $\bar{G}$  is non planar. 10+10

Section-C

- V. (a) Find the particular solution of the difference equation :  $a_r - 4a_{r-1} + 4a_{r-2} = (r+1)2^r$   
 (b) Solve the recurrence relation  $a_{r+2} - 3a_{r+1} + 2a_r = 0$  by the method of generating functions with the initial conditions  $a_0 = 2$  and  $a_1 = 3$ . 10+10

- VI. (a) Solve  $a_r - 4a_{r-1} + 3a_{r-2} = r^2$   
 (b) Find the generating function from the recurrence relation  $a_r + a_{r-1} - a_{r-2} = 0$  with the initial conditions  $a_0 = 1$  and  $a_1 = 1$ . 10+10

Section-D

- VII. (a) Consider an algebraic system  $(G, *)$ , where  $G$  is the set of all non zero real numbers and  $*$  is a binary operation defined by  $a * b = \frac{ab}{4}$ . Show that  $(G, *)$  is an abelian group.

- (b) Consider the lattice  $D_{60} = \{1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60\}$ :  
 (i) Draw the Hasse Diagram of  $D_{60}$  (ii) Find all Join-irreducible elements  
 (iii) Find all atoms (iv) Find complements of 2 and 10, if they exist.  
 (v) Express each  $x$  as a join of irredundant join-irreducible elements. 10+10

- VIII. (a) If  $f(x, y, z) = f(x, y, z) = (x \vee y) \wedge (x \vee y') \wedge (x' \vee z)$  be the given Boolean function. Determine its Disjunctive Normal form.

- (b) Design the combinatorial circuit for the Boolean expressions  $(x \vee y') \vee (x' \vee y)$  and  $(x \cdot y + z) \cdot (y + z) + z$

- (c) Show that the Boolean expressions  $x \wedge (y \vee (y' \wedge (y \vee y')))$  and  $x$  are equivalent. 7+7+6

Section-E

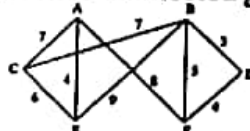
IX. Do as directed :

- (a) Prove that  $\frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{2n} > \frac{13}{24}$ , for  $n \geq 2$ .

- (b) Let  $R$  be an equivalence relation on the set  $A = \{6, 7, 8, 9, 10\}$  defined by  $R = \{(6,6), (7,7), (8,8), (9,9), (10,10), (6,7), (7,6), (8,9), (9,8), (9,10), (10,9), (8,10), (10,8)\}$ . Find the equivalence classes of  $R$  and hence find the partition of  $A$  corresponding to  $R$ .

- (c) Show that at least two people must have their birthday in the same month if 13 people are assembled in a room.

- (d) Find a minimum spanning tree of the labelled connected graph.



- (e) Prove that in a simple graph with  $n$  vertices, each vertex has maximum degree  $(n-1)$

- (f) Show that  $K_{n-1}$  is non planar graph

- (g) Let  $a = 3^r$ , and  $b = 2^r$ , for  $r > 0$ . Then find  $c$  that is  $a * b$ .

- (h) What is the meaning of Disjunctive Normal form of a Boolean expression ?

- (i) State and prove absorption laws for Boolean algebra.

- (j) Let  $D_{100} = \{1, 2, 4, 5, 10, 20, 25, 50, 100\}$  and let the relation  $<$  be the partial ordering relation on  $D_{100}$ . Find g.l.b. and l.u.b. of  $\{10, 20\}$  and  $\{5, 10, 20, 25\}$  2x10=20