DISCRETE MATHEMATICS-I

Time : Three Hours] [Maximum Marks : 40

Note : Attempt *two* questions each from Section A and B carrying 7½ marks each. Section C is compulsory consisiting of 10 short answer type questions carrying 1 mark each.

Section - A

I. Let $f: X \to Y$ and $g: Y \to Z$ and let f, g be one-one onto. Then $gf: X \to Z$ is alos one-one onto and $(g \circ f)^{-1} = f^{-1} \circ g^{-1}$. 7½

II. Let A = [1, 2, 3, ...9] and '~' be a relation on A × A defined by (a, b) iff a+ b = b + c.

(i) Prove that '~' is an equivalence relation.

(ii) Find the equivalence clas of (2,5) 7½

III. State principle of Inclusion and Exclusion. In a class of 25 students, 12 have taken economics, 8 have taken economics but not political science. Find the number of students who have taken economics and political science and those who have taken political science but not economics. 7½

IV. Find the minimum number n of integers to be selected from S = {1, 2, 3, ...9} so that

(i) the sum of two of the n integers is even.

(ii) the difference of two of the n integers is 5. 7½

Section - B

V. State Euler's Theorem. Prove that the number of edges in a complete graph with a vertices is $\frac{m(n-1)}{2}$ 7½

VI. For a graph G prove that

(i) G is connected iff it has a spanning tree.

(ii) G is a tree ift it is minimally connected. 7½

VII. Produce a finite state machine that adds two integers using their binary expansions. 7½

VIII. let G = (v, E) be a conntected planar graph and let R be the number of regions defined by any planar depiction of G. Then R = |E| - |V| + 2. 7½

Section - C

(Compulsory Questions)

IX. Answer in brief :

(a) Give Check whether a relation "divides" on a set of integers is a partial order relation ?

(b) Define regular grammar and Context free grammar.

(c) Draw the Hasse diagram of the relation $\subseteq$ on P(A), where A = {a, b, c}.

(d) Prove that there is one and only one path between every pair of vertices in a tree.

(e) The function f(x) = 2x, $x \geq 0$ has an inverse ? Give specific answer.

(f) State Travelling Salesman Problem.

(g) Find n, if a complete graph having n vertices has 15 edges.

(h) Define d-regular graph. Draw 3-regular graph with eight vertices.

(i) Prove that the graph K_5 is not planar.

(j) What is pigeonhole principle and extended pigeonhole principle? 1x10=10