ALGEBRA – II

Paper - III
Semester-VI

Time Allowed : 3 Hours]　　　　　　　　　　　　　　　　[Maximum Marks : 36

Note : The candidates are required to attempt *two* questions each from Sections A and B carrying 5.5 marks each and the entire Section C consisting of 10 short answer type questions carrying 2 marks each.

Section - A

1.   (a)   Show that any plane passing through (0, 0, 0) is a subspace of R^3.   3
     (b)   Examine whether $\{1, -3, 5\}$ belongs to the linear space generated by S, where S = $\{1, 2, 1) (1, 1, -1), (4, 5, -2)\}$ or not ?   2.5

2.   (a)   Let V be a vector space of function $f : R \rightarrow R$. Then show that all f where $f(-2) = 0$ is a subspace or not of V ? Justify your answer.   3
     (b)   State and prove Replacement theorem.   2.5

3.   (a)   Let V be vector space of 2 ×2 matrices of R and W be asset of all 2 ×2 diagonal matrices over R. Show that W is a subspace of V and find basis of V/W.   2.5
     (b)   If V and W are fine dimensional subspaces of a fine dimensional vector space U(F). Prove that V + W is also finite dimensional and dim (V + W) = dim V + dim W – dim V $\cap$ W.   3

4.   (a)   Extend $\{(-1, 2, 5)\}$ to two different basis of R^3.   2.5
     (b)   Find basis and dimension of the solution space W of the following system of equations :
$$x + 2y - 4z + 3t - s = 0,$$
$$x + 2y - 2z + 2t + s = 0,$$
$$2x + 4y - 2z + 3t + 4s = 0.$$
  3

Section - B

5.   (a)   Find linear transformation $T : R^2 \rightarrow R^2$ such that T (2, 3) = (1, 2) and T (3, 2) = (2, 3).   2.5
     (b)   Prove that non-zero eigen vector corresponding to distinct eigen values of a linear operator are linearly independent.   3

6.   (a)   Find range, rank null space and nullity for T(x, y) = (x + y, x – y, y) transformation on a vector space R^2.   3
     (b)   Find a linear map $T : R^4 \rightarrow R^3$ whose will null space is generated by (1, 2, 3, 4) and (0, 1, 1, 1).   2.5

7.   Find characteristic and minimal polynomial of a matrix $\begin{bmatrix} 6 & -3 & -2 \\ 4 & -1 & -2 \\ 10 & -5 & -3 \end{bmatrix}$, using idea of minimal polynomial, find inverse of matrix.   5.5

8. (a) if the matrix of the linear operator T or R^3 relative to basis :

$B = \{(e_1, e_2, e_3)\} = \{(1, 0, 0)\ (0, 1, 0),\ (0, 0, 1)\}$ is $\begin{bmatrix} 0 & -1 & -1 \\ -1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$. Find the matrix of T

relative to basis
$B_1 = \{(0, -1, 1), (1, -1, -1), (1, -1, 0)\}$.   2.5

(b) Prove that two finite dimensional vector space V(F) and W(F) over the same field F are isomorphic iff they have same dimensions.   3

Section - C

9. (a) Show that all polynomials over R with constant terms 2 doesn't from vector space.

(b) If S is a subspace of V then show that L(s) = S.

(c) Find value of K so that the vector (1, –2, k) becomes linear combination of the vectors (3, 0, –2) and (2, –1, –5).

(d) If S is a subset of V, then prove that S– = span $(S)^{\perp}$.

(e) State and prove Pythagoras theorem for orthogonal set of vectors.

(f) Prove that minimal polynomial of a matrix exist uniquely.

(g) Define singular transformation and Nullity of transformation.

(h) Show that set containing vector 0 is always linear dependent.

(i) The minimal polynomial of an operator T divides every polynomial which has T as a zero.

(j) Prove that if α is an eigen value of a linear operator T on V(F) iff α is a root of the minimal polynomial of T.   10×1.4=14