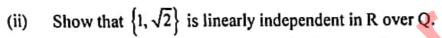
ALGEBRA-II

Paper-III : Semester-VI

Time Allowed: Three Hours Maximum Marks:		
Note: The candidates are required to attempt two questions each from Section A and B carrying 71/2		
marks each and the entire Section C consisting of 10 short answer type questions carrying 1 marks each.		
		SECTION-A
1.	If U	and W are subspaces of a finite dimensional vector space over F, then prove that
	dim($U + W) = \dim(U) + \dim(W) - \dim(U \cap W). $ 7½
2.	(a) `	Show that the set of all real valued continuous functions $y = f(x)$ satisfying the
		differential equation y" + 6y" + 11y' + 6y = 0 is a vector space over R. Find also the
		basis of the vector space.
	(b)	Under what condition on scalar τ do the vectors (1, 1, 1), (1, τ , τ^2) and (1, $-\tau$, τ^2)
	` ,	forms a basis of C ³ ?
3.	(a)	If x, y and z are vectors in vector space over F such that $x + y + z = 0$, then show
	3 5	that x and y span the same subspace as y and z.
	(b)	Let V be a vector space over an infinite field. Prove that V can not be written as set
	` '	theoretic union of a finite number of proper subspaces. 4
4.	(a)	Find the matrices of ordered basis (f ₁ , f ₂ , f ₃) relative to standard basis (e ₁ , e ₂ , e ₃) of
		vector space R ³ if $f_1 = (1, \cos x, \sin x)$, $f_2 = (1, 0, 0)$ and $f_3 = (1, -\sin x, \cos x)$. 3½
*2	(b)	Prove that every subspace of a finite dimensional vector space has a complement.
		4
		SECTION-B
5.	(a)	Show that a linear transformation $T: V \rightarrow V$ is singular if and only if its minimal
		polynomial has nonzero constant term.
	(b)	Let T be a linear operator on V and $Rank(T^2) = Rank(T)$, then show that the Range
		$(T) \cap \text{Ker}(T) = (0).$ 3½
6.	(a)	State and prove Sylvester's law of nullity.
	(b)	Let T: R ⁴ \rightarrow R ³ be a linear transformation defined by T (x, y, z, t) = $(x - y + z + t,$
	2x -	2y + 3z + 4t, $3x - 3y + 4z + 5t$), then find a basis and dimension of the image of T.
		314
7.	(a)	Let T be a linear operator on n dimensional space V, then show that the characteristic
	00.20.2	and minimal polynomials for T have the same roots.
2	(b)	Prove that a linear transformation T is one-one if and only if $Ker(T) = (0)$. $3\frac{1}{2}$
8.	Let U	and V be two vector spaces over the same field F dimensions n and m respectively.
	then s	show that homomorphism from U to V is isomorphic to the vector space of all m x
	n mai	trices over F.
_	3 A	SECTION-C
9.	(i)	Show that any n + 1 member of vector space V of dimension n are linearly dependent.
		• • • • • • • • • • • • • • • • • • • •



- Define the Quotient space with suitable example. (iii)
- Find the value of k so that the vectors (1, -1, 3), (1, 2, -2) and (k, 0, 1) are linearly (iv)
- Determine the complement of the subspace of V generated by $\{(1, 1, 0), (0, 1, 0)\}$. (v)
- Check whether a mapping $T: \mathbb{R}^3 \to \mathbb{R}$ defined by $T(x, y, z) = x^2 + y^2 + z^2$ is a linear (vi) transformation.
- Extend the set $S = \{(1, 1, 1)\}$ as a basis of \mathbb{R}^3 . (vii)

- Show that E_1E_2 is a projection if $E_1E_2 = E_2E_1$ where E_1 and E_2 are projections. Give an example of a Linear transformation T on R^2 such that $T^2 = 0$ but $T \neq 0$. (ix)
- Find the rank of the linear transformation $T: \mathbb{R}^2 \to \mathbb{R}^3$ such that T(x, y) =(x) (x, x + y, y).