ADVANCED CALCULUS – I

Time : Three Hours **Maximum Marks : 100**

Note : Attempt one question each from Sections A, B, C and D and the entire Section E. All questions carry equal marks.

SECTION–A

I. (a) State and prove Cantor Intersection Theorem.

(b) Prove that the sequence (x_n), where $x_n = \frac{2n-5}{3n+1}$ is a monotonically increasing and bounded above as well as below. 20

II. (a) The image of closed interval under a continuous function is a closed interval.

(b) Show that $f(x) = \frac{1}{x-a}$ is discontinuous at $x = a$. Locate the type of discontinuity. 20

SECTION – B

III. Expand the following using Maclaurin's Theorem with Lagrange's form of Remainder :

(a) $\cos x$ (b) $\sin x$ 20

IV. (a) State and prove First Mean Value Theorem.

(b) Let $f : R^2 \rightarrow R$ be defined by

$$f(x,y) = x\sin\frac{1}{y} + y\sin\frac{1}{x}, x, y \neq 0$$

Prove that $\underset{(x,y)\to(0,0)}{\text{LT}} f(x,y) = 0$ 20

SECTION–C

V. (a) State and prove Taylor's Theorem for function of two variables.

(b) Discuss the differentiability of the function

$f(x, y) = (xy)^{1/3}$ at $(0, 0)$. 20

VI. Find the maximum and minimum values of

$u = \sin x + \sin y + \cos (x + y)$. 20

SECTION – D

VII. (a) Test the nature of the following series :

$$\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n^p}, p>0.$$

(b) State and prove Ratio Test. 20

VIII. (a) State and prove Dirichlet's Test.

(b) Test the convergence of the series $\sum_{n=1}^{\infty}\frac{1}{n^2+1}$ 20

SECTION – E

IX. Do as directed :

(a) Prove that $f(x) = 9x + 1$ is uniformly continuous on [2, 3].

(b) If $f(x)$ is continuous in [a, b], then it is bounded in [a, b].

(c) Define Bounded and Monotonic sequence with examples.

(d) State Taylor's Theorem.

(e) Define Partial Differentiation with example.

(f) State Euler's Theorem on homogenous function.

(g) Show that the function $f(x, y) = \cos x + \cos y$ is differentiable at every point.

(h) Find the maximum or minimum of $u = 2(x - y)^2$.

(i) State Schwarz's Theorem.

(j) Define Saddle point of function of two variables.

$2\times10=20$