ANALYSIS-II

Time: Three Hours
Note: The candidates are required to attempt one question each from Sections A, B, C and D carrying 20 marks each and the entire Section E Consisting of 8 short answer type questions carrying 2½ marks each.

Section-A

- 1. (a) State and prove Darboux's Theorem.
 - (b) Let f be a function defined on [0, p/4] by $[\sin x, \text{ if } x \text{ is irrotational}]$

 $f(x) = \begin{cases} \sin x, & \text{if } x \text{ is irrotational} \\ \cos x, & \text{if } x \text{ is rotational} \end{cases}$

Show that f is not Riemann Integrable.

Let f be Riemann Integrable on [a, b] and g be another function obtained by altering the value of f at a finite number of points of [a,b], then prove that g is also Riemann Integrable

on [a, b] and $\int_{a}^{b} f dx = \int_{a}^{b} g dx$

7+7+6

- 2 (a) If a function f defined on an interval [a, b] is bounded and has a finite points of discontinuity then prove that f is Riemann Integrable.
 - (b) The necessary and sufficient condition for a bounded function f to be Riemann Integrable on an interval [a, b] is that to every $\hat{I} > 0$, however small, there exists a partition P such that U(P, 1) L(P, f) < I
 - (c) Evaluate $\int_{0}^{\pi} \sin x \, dx$ by calculating upper and lower integrals.

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Section-B

3. (a) Prove that $\lim_{n\to\infty} \sqrt[n]{a=1}$ for all a>0

- Prove that the sequence $\left\{ \left(1 + \frac{3}{n}\right)^n \right\}$ is monotonically increasing and bounded, also (b) show that it converges to the limit e³. Prove that the sequence $[x_n]$ converges to a real number 1 iff $[x_n]$ is bounded and 1 is the only cluster point of $[x_n]$. (c) State and prove Leibnitz test. (a) Using Cauchy's Integral test, discuss the convergence of the series (b)
 - $\sum_{n=1}^{\infty} \frac{1}{n^p}, \quad p > 0$

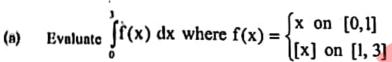
Discuss the convergence of the series. (c) $\sum_{n=0}^{\infty} \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \dots + \frac{1}{n}\right) \sin(n\theta + a)$

Section-C

- Test the sequence $\left| \frac{n^2 x}{1 + n^3 x^2} \right|$ for uniform convergence on the interval (0, 1) 5. (a)
 - Examine for term by term integration the ries (b)
 - If $na_{n \otimes 0}$ and $f(x) \sum a_n x^n \to s$ as $x \to 1$, then $\sum a_n$ converges to the sum S. (c)
- State and prove Abel's test. 6. (a)
 - Show that $\tan^{-1}x = x = \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \dots$ for -1 £ x £ 1. (b) Also deduce that series $\frac{p}{4} = 1 - \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \dots$
 - Test the series $\sum nx(1-x)^4$, $0 \le x \le 1$ for uniform convergence and continuity of the sum function for all values of x. (c)
- Find all the circumcentre of the triangle whose vertices are complex numbers z₁, z₂, z₃. 7. (a)
 - Find all bilinear transformation which transform the plane $I_m(z)^3$ 0 into the unit circle **(b)**
 - If f(z) = u + iv is an analytic function of z = x + iy and $u v = e^x$ (cos y sin y), find f(z) in (c) terms of z.
- Show that locus of complex number z such that $|z-a|z+a|=a^2$, a>0 is a lemiscate. (a)
 - Let f(z) be an analytic function of z in a region D of the z-plane and let $f'(z) \neq 0$ inside (b) D. Then the mapping w = f(z) is conformal of the points of D.
 - Show that $w = \sinh z$ satisfies Cauchy-Riemann equations and find $\frac{dz}{dz}$ (c)

Section-E

Do as directed:



- Prove that $1 \le \int_{0}^{\pi} \sqrt{\sin x} \, dx \le \frac{1}{2} \sqrt{2\pi}$ (b)
- Give an example to show that $\{x_n, y_n\}$ can be convergent without $\{x_n\}$, $\{y_n\}$ being (e) convergent.
- Examine the convergence of the series $\sum_{n=1}^{\infty} \frac{n^2}{n!}$ (d)
- Find the radius of covaergence and interval of convergence of the series $\sum \frac{(x-1)^n}{2^n}$ (e)
- What is the difference between Pointwise convergence and Uniform convergence?
- (f) (g) Show that $f(z) = z^2$ is uniformly continuous in the region.

(h) Show that
$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) f = 4 \frac{\partial^2 f}{\partial z \partial \overline{z}}$$