DIFFERENTIAL EQUATIONS-II

Time : Three Hours **Maximum Marks : 100**

Note : Attempt one question each from Sections A, B, C and D carrying 20 marks each, and, the entire Section E is compulsory consisting of 10 short answer questions carrying 2 marks each.

SECTION–A

I. (a) Solve in series $x^2\frac{d^2y}{dx^2}+x.\frac{dy}{dx}+(x^2-4)=0$. (10)

(b) Prove that $e^{\frac{x}{2}\left(t-\frac{1}{t}\right)}$ is a generating function of Bessel's function. (10)

II. (a) Find the Eigenvalues and Eigenfunctions of Sturm Liouville's problem $\frac{d^2y}{dx^2}+\lambda y=0$

$y(0)=0$ and $y(\alpha)=0$ (10)

(b) Prove that

$$\frac{d}{dx}F(l, m, p; x)=\frac{lm}{p}F(P+1, m+1, p+1; x).$$ (5)

(c) Express $5x^2-3x+6$ in terms of Legendre's polynomial. (5)

SECTION – B

III. (a) Find the general solution of partial differential equation $r-q=e^{x+y}$. (10)

(b) Solve for complete solution $(p^2+q^2)\,y=qz$ by Charpit's method. (10)

IV. (a) Solve the partial differential equation

$$\frac{\partial^2 z}{\partial x^2}-\frac{\partial^2 z}{\partial y^2}+\frac{\partial z}{\partial x}+3\frac{\partial z}{\partial y}=2z+e^{x-y}-x^2y.$$ (10)

(b) Solve $p^2+q^2-2px-2qy+1=0$. (10)

SECTION–C

V. (a) Find the extremal of the functional $\int_0^{\frac{\pi}{2}}(y'^2+z'^2+2yz)\,dx$. Given that $y(0)=0$,

$y\left(\frac{\pi}{2}\right)=1,\ z(0)=0,\ z\left(\frac{\pi}{2}\right)=-1$ (10)

(b) Apply Convolution theorem to find Inverse Laplace transformation of $\frac{10}{(s-3)(s+7)}$. (5)

(c) Solve $\frac{d^2y}{dx^2}+\frac{dy}{dx}=2$, where $y(0)=3$, $y'(0)=1$ by using Laplace transformation. (5)

VI. (a) Solve the integral equation

$$y(t)=e^{-t}-2\int_0^t y(u)\,.\cos(t-u)\,du$$ (10)

(b) Find the extremal of $\int_0^1 yy'^3dx, y(0)=0, y(1)=1$ (5)

(c) If $L(f(t))=\bar{f}(s), t\geq 0$ and α is any no. real or complex.
Prove that if function g (t).

Where $g(t)\begin{cases} f(t-x), & t>\alpha \\ 0, & t<\alpha \end{cases}$

then $L(g(t))=e^{-\alpha s}\,\bar{f}(s)$ (5)

SECTION – D

VII. (a) Find the shortest distance from the point P (1, 2, 2) to the sphere $x^2+y^2+z^2=1$. (10)

(b) Find the extremal of $\int_a^b (x^2y'^2+2y^2+2xy)\,dx$. (10)

VIII. (a) Find the extremum of given functional $J[y(x)]=\int_4^5 \frac{x^2}{(y')^3}dx$. Given that $y(4)=64$, $y(5)=125$ by using Legendre's condition. (10)

(b) Show that Jacobi Condition is fulfilled for the functional $\int_0^c (x^2+y'^2)\,dx$ with fixed boundaries P(0, 0) and Q (C, 2) C > 0. (10)

SECTION – E
(Compulsory Question)

IX. (i) Show that for $-1<x<1$, $F(1, 1, 1; x)=\frac{1}{1-x}$.

(ii) Solve $r+s=0$.

(iii) Find the solution of

$$\frac{\partial^3 z}{\partial x^3}-6\frac{\partial^3 z}{\partial x^2\partial y}+11\frac{\partial^3 z}{\partial x\partial y^2}-6\frac{\partial^3 z}{\partial y^3}=0.$$

(iv) Find the differential equation of right circular cone having vertex at (0, 0) and axis as Z–axis.

(v) State First Shifting theorem of Laplace transformation.

(vi) Define Singular solution.

(vii) Form p.d.e. by eliminating arbitrary constants from the following $z=(2x+a)(2y+b)$.

(viii) Solve $(z^2-2yz-y^2)p+(xy+zx)q=xy-zx$.

(ix) State sufficient conditions for Extremum of Jacobi condition.

(x) Prove that $J^2_{1/2}(x)+J^2_{-1/2}(x)=\frac{2}{\pi x}$.

2×10=20