

MECHANICS - III

Time : Three Hours

Maximum Marks : 100

Note :- Attempt one question each from Section A, B, C and D carrying 20 marks each and the entire Section A consisting of 10 short answer type questions carrying 2 marks each.

Section-A

1. (a) Find the components of a given force along two given directions. 10
 (b) The resultant of two forces P and Q trisects the angle between them. Show that if $P > Q$, the angle between them is $3 \cos^{-1} \left(\frac{P}{2Q} \right)$ and magnitude of results is $\left(\frac{P^2 - Q^2}{Q} \right)$ 10
2. (a) Forces of magnitude P, 2P and 3P act along the sides BC, CA and AB of an equilateral ΔABC . Find their resultant in magnitude, direction and line of action. 10
 (b) ABCD is a rectangle with sides AB and BC of a and b units respectively. Forces P, P act along AB and CD and forces Q, Q act along AD and CB ($P > Q$). Prove that the perpendicular distance between the resultant of forces P, Q acting at A and resultant of forces P, Q acting at C is $\frac{P \cdot b - Q \cdot a}{\sqrt{P^2 + Q^2}}$ 10

Section-B

3. (a) State and prove Lami's Theorem. 10
 (b) A string ABCD is suspended from two fixed points A and D. It carries a weight of 30kg and Wkg respectively at two points B and C on it. the inclination to the vertical of AB is 30° and that of CD is 60° , the angle BCD being 120° . Find W and the tension in different parts of the string. 10
4. (a) Prove that the horizontal force which will just sustain a heavy particle on rough inclined plane will sustain the same particle on smooth plane whose inclination is less than that of the rough plane by the angle of friction. 10
 (b) How high can a particle rest inside a rough hollow sphere of radius a if the coefficient of friction is μ ? 10

Section-C

5. (a) A bus is beginning to move with an acceleration of $\frac{1}{3} m/sec^2$. A man who is $13\frac{1}{3} m$

- behind the bus starts running at 3 m/sec to catch the bus. After what time will the man be able to catch the bus? Explain the double answer. 10
- (b) A block of 80 kg is placed on an incline of 3 in 5. Find the horizontal force required to produce in the block an acceleration of 4 m/sec² up the plane. Find also the pressure on the plane. 10
6. (a) At the end of three successive seconds, the distances of a point moving with S.H.M. from its mean position measured in the same direction are 1, 5, 5. Show that the period of complete oscillations is $\frac{2p}{q}$ secs. Where $\cos q = \frac{3}{5}$. 10
- (b) A ball is projected so as just to clear two walls the first of height a at distance b units from the point of projection and second of height b at a distance of a units from the point of projection. Show that the range on the horizontal plane is $\frac{a^2 + ab + b^2}{a + b}$ and that the angle of projection exceeds $\tan^{-1}3$. 10
- Section-D**
7. (a) Prove that work done by a couple is equal to product of moment of the couple and circular measure of the angle of rotation. 10
- (b) A pendulum which beats seconds at the surface of earth is carried to the top of a mountain 5km high. How many seconds will it lose or gain per day? What correction in its present length be made so that it may beat seconds at the top of the mountain? 10
8. (a) A ball of mass 5gm falls from a height of 10m and rebounds to a height of 6.4m. Find the impulse and the average force between the ball and the ground if the impact takes place for $\frac{1}{50}$ seconds. 10
- (b) A train moving at the rate of 50 km/hr is struck by a stone moving with velocity of 40 km/hr making an angle of 60° with the direction of train. Find the velocity with which the stone appears to an observer in the train to strike the train. 10
- Section-E**
9. Do as directed :
- (a) State $I - m$ Theorem. 2
- (b) Two forces act at an angle of 120°. The greater is represented by 80 kg and the resultant is right angle to the least. Find the least force. 2
- (c) State theorem on resolved parts. 2
- (d) Give geometrical representation of the movement of a force about a point. 2
- (e) State laws of friction. 2
- (f) A man weighing 62 kg is in lift which is descending with an acceleration of 0.7 m/sec². Find the thrust of his feet on the lift. 2
- (g) Show that S.H.M. is a periodic motion. 2
- (h) Define Projectile and trajectory. 2
- (i) Find Potential energy of a gravitational field. 2
- (j) Define escape velocity. 2