ADVANCED CALCULUS

Paper- I Semester-III

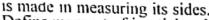
Time Allowed: 3 Hours]

[Maximum Marks : 36

Note: The candidates are required to attempt two questions each from Section A and B carrying 5.5 marks each and the entire Section C consisting of 7 short answer type questions carrying 2

- If $u = x^2 \tan^{-1} \left(\frac{x}{y} \right) y^2 \tan^{-1} \left(\frac{x}{y} \right)$, then evaluate $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$. ŀ.
 - u = f(r) where $r^2 = x^2 + y^2$, show that : $\frac{\partial^2 \mathbf{u}}{\partial \mathbf{x}^2} + \mathbf{y}^2 \frac{\partial^2 \mathbf{u}}{\partial \mathbf{v}^2} = \mathbf{f}(\mathbf{x})^n = \frac{1}{\mathbf{r}} \mathbf{f}'(\mathbf{r}).$
- 2. Show that the function f(x, y) = |x| + |y| is continuous at (0, 0), but not differentiable at (0, 0)
 - State and prove Taylor's theorem for functions of two variables
- If $u = \tan^{-1} \left(\frac{x^3 + y^3}{x y} \right)$, prove that 3. $x^{2} \frac{\partial^{2} u}{\partial x^{2}} + 2xy \frac{\partial^{2} u}{\partial x \partial y} + y^{2} = \frac{\partial^{2} u}{\partial y^{2}} = \left(1 - 4\sin^{2} u\right) \sin 2u.$
- Show that $f(x, y) = \sin x + \cos y$, is differentiable at every point of R^2 . Find the minimum value of $x^2 + y^2 + z^2$, when $yz + zx + xy = 3a^2$, where a is constant. 4.
 - Find $\frac{\partial (y_1 y_2 \dots y_n)}{\partial (x_1 x_2 \dots x_n)}$, it is being given that: (b) $y_1 = x_1 (1-x_2), y_2 = x_1x_2 (1-x_3), y_3 = (x_1x_2x_3) (1-x_4)$ and $y_{n-1} = x_1x_2x_3x_4$ $x_{n-1(1-x_0)}, y_1 = x_1x_2x_3x_4$ x_n .

 Section - B
- Evaluate by changing the order of integration of: $\int_{0}^{1/2-x^2} \frac{x \, dy \, dx}{\sqrt{x^2+y^2}}$ 5.
 - Evaluate $\iiint (x^2 + y^2) dx dy dx$ over the region bounded by $x^2 + y^2 + z^1 = 1$. (b)
- Evaluate $\iint (a^2 x^2 y^2) dx dy$ over the semi circle $x^2 + y^2 \le ax$ in the first quadrant, a > 0. (a)
- Using triple integration, find the volume of a sphere $x^2 + y^2 + z^2 = a^2$, where a is constant. Using double integration, find the area of the region enclosed by the curves $y^2 = x^3$ and $y = x^3$ 7.
 - Find the centriod of an ellipse with mass density 1.
- Find the moment of inertia of the ellipsoid
 - $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{a^2} \le 1$. With uniform mass density 1 about the y-axis.
 - Find the area of the circle of radius r by using polar co-ordinates. (b) Section - C
- Using definition prove, that $\lim_{(x,y)\to(1,2)} (3xy + 5x 4) = 7$. (a)
 - Find the percentage error in calculating the area of a rectangle when an error of 2 percent (b)



- is made in measuring its sides.

 Define moment of inertial and centre of gravity in R³. (c)
- Show that : $2 \le \iint_{1 \le x \le 2} (x^2 + y^2) dx dy \le 8$. (d)
- Evaluate $\iint_E \sin\left(\frac{x-y}{x+y}\right) dx dy$, where E is the region bounded by the coordinate axes (e)

- and x + y = 1 int he first quadrant. (f) Prove that u = x 2y + z, $v = x^2 + 2xy xz$, z = 2 = 3x + 2y z are not functionally
- State Euler's theorem of Homogeneous functions. Also define Homogeneous functions. (g)