ANALYSIS – I

Paper- II
Semester-III

Time Allowed : 3 Hours] [Maximum Marks : 36

Note : The candidates are required to attempt **two** questions each from Section A and B carrying 5.4 marks each and the entire Section C consisting of 10 short answer type questions carrying 1.4 marks each.

Section - A

1. (a) Prove that every monotonic function defined on a closed interval is Riemann integrable.
   (b) If f be a function defined on [0, 2] by

$$f(x)=\begin{cases} x^2+x, & \text{if } x \text{ is rational} \\ x^3+x^2, & \text{if } x \text{ irrational} \end{cases}$$, show that f is not R-integrable. 3

2. (a) Evaluate $\int_0^{\frac{\pi}{2}} \log \sin x \, dx = -\frac{\pi}{2}\log 2$ 2.5

   (b) Prove that $\frac{\sqrt{3}}{8} \le \int_{\frac{\pi}{4}}^{\frac{\pi}{3}} \frac{\sin x}{x} dx \le \frac{\sqrt{2}}{6}$. 3

3. (a) Use method of summatio to evaluate $\int_a^b \frac{1}{x} dx$. 3

(b) State and prove Darbous's theorem. 2.5

4. (a) If $f \in [a, b]$ and if is continuous at point $x_0 \in [a,b]$, then

$F'(x_0) = f(x_0)$ where $F(x) = \int_a^x f(t)dt$.

(b) Let (fx) = 3x + 1 on [1, 2]. Prove that f is R-integrable on [1,2] and $\int_1^2 f(x)dx = \frac{11}{2}$. 3

Section - B

5. (a) Find directional derivative of ∇^2 where $\bar{\nabla} = 2xy^2\hat{i} + zy^2\hat{j} + xz^2\hat{k}$ at a point (2, 0, 3) in the direction of outward normal to the surface $x^2 + y^2 + z^2 = 14$ at a point (3, 2, 1). 2.5

(b) Show that curl $\vec{v} = \text{grad div}\,\vec{v} - \nabla^2\vec{v}$ where $\vec{v}$ is any vector. 3

6. (a) If $\vec{f} = (2x^2 - 3x)\hat{i} - 2xy\hat{j} - 4x\hat{k}$, then evaluate $\iiint_V \vec{f}\,dv$ where V is the region bounded by the plane $2x + 2y + z = 4$. 2.5

(b) Evaluate $\int_C \phi\, d\bar{r}$ for $\phi = x^2y + 2y$ along the line joining (1, 1, 0) to (2, 4, 0) 3

7. (a) If $\vec{f} = (2x^2 + y^2)\hat{i} + (3y - 4x)\hat{j}$, evaluate $\int_C \vec{f}\, d\vec{c}$ around the triangle ABC whose vertices are A(0, 0), B (2, 0) and C(2, 1). 3

(b) Verify divergence theorem for $\vec{f} = 4x\hat{i} - 2y^2\hat{j} + z^2\hat{k}$ taken over the fegion bounded by cylinder $x^2 + y^2 = 9$, $z = 0$, $z = 4$.

8. (a) State and prove Green's theorem. 2.5

(b) Evaluate $\oint_C (xydx + xy^2dy)$ by stoke's theorem where C is the square in the XY-plane with vertices (0, 1), (−1, 0), (1, 0), (0, −1). 3

Section - C

9. (i) State necessary and sufficient condition for a function to be Reimann integrable.

(ii) Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} |\sin x|\,dx$.

(iii) State first mean value theorem.

(iv) Compute L (P, f) for f = sin x where $P = \left\{0, \frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{2}\right\}$

(v) Prove that $\int_0^a f(x)dx = \int_0^a f(a - x)dx$.

(vi) The necessary and sufficient condition for $\vec{f}(t)$ to have contant magnitude is $\vec{f}.\frac{d\vec{f}}{dt} = 0$.

(vii) If $\vec{A} = 2z\hat{i} + y\hat{j} - x^2\hat{k}$, $\vec{B} = x^2yz\hat{i} - 2xz\hat{i} - 2xz^3\hat{j} - xz^2\hat{k}$ then find $\frac{\partial^2}{\partial x \partial y}(\vec{A} \times \vec{B})$ and (1, 1,1).

(viii) If $\vec{f}$ is solenoidal vector then show that curl curl curl curl $\vec{f} = \nabla^4 f$.

(ix) Prove that $\int \bar{r}.dr = 0$, where r has its usual meaning.

(x) Show that the function $1/r$ is harmonic, where r has its usual meaning. $10 \times 1.4 = 14$