STATICS – III

Semester – III

Time Allowed : 3 Hours] [Maximum Marks : 36

Note : The candidates are required to attempt two questions each from Section A and B carrying 5½

marks each and the entire Section C consisting of 7 short answer type questions carrying 2 marks each.

Section - A

1 (a) ABCD is a rectangle with sides AB = 5 meters and BC = 12 meters. Along the sides AB, BC, CD, DA and diagonal AC, act forces equal to 9, 8, 24, 44 and 39 kilograms respectively. Show that they are equivalent to a couple and find the moment of the couple.

(b) Two forces of magnitudes P + Q and P – Q make an angle 2α with one another, and their resultant makes an angle θ with the bisector of the angle between them. Show that $P \tan \theta = Q \tan \alpha$. 3,2½

2. (a) The resultant R of two forces P and Q act at right angles to P; show that the angle between the forces is $\cos^{-1}\left(-\frac{P}{Q}\right)$ and $Q^2 - P^2 = R^2$.

(b) Forces of magnitude P, 2P, –3P, 4P, 5P, and –9P act respectively along the sides taken in order of a regular hexagon. Prove that they reduce to a single resultant force acting through the centre of the hexagon. Find the magnitude and direction of their resultant. 3,2½

3. (a) ABCDEF is a regular hexagon and force represented in magnitude and direction by the segments, $\overline{AB}$, $\overline{AC}$, $\overline{AD}$, $\overline{AE}$, $\overline{AF}$ act at the point A. Find the magnitude and direction of the their resultant.

b) Two forces equal to P and 2P in magnitude act on a particle. If the first be doubled and the second increased by 10 kg., the direction of the resultant is unaltered. Find the value of P. 3,2½

4. (a) Forces each of magnitude P act at a point in directions parallel to the sides BC, CA, AB of ΔABC. Prove that the magnitude of their resultant is $P\sqrt{3 - 2\cos A - 2\cos B - 2\cos C}$.

(b) A heavy uniform rod 200 cm. long, rests horizontally on two pegs which are 50 cm. apart; a weight of 20 kg. suspended from one end or a weight of 8 kg. suspended form the other end will just tilt the rod up; find the weight of the rod. 3,2½

Section - B

5. (a) A string ABCD is suspended from two fixed points A and D. It carries weight of 30 kg. and W kag. respectively at two points P and C in it. The inclination of AB to vertical is 30° and that of CD to the vertical is 60°, the angle BCD being 120°. Find W and the tension in different part of the string.

(b) Two strings of length 4 meters and 5 meters are fastened to a particle of mass W kg., their other ends being fastened to points at the same level 6 meters apart, find the tensions in the strings. 3,2½

6. (a) A rod whose length is equal to the radius of a hollow sphere is placed inside the sphere. Its centre of gravity is at a distance one-third of its length one end. Show that the inclination of the rod to the horizontal in the position of equilibrium is $\cot^{-1}\left(3 - \sqrt{3}\right)$ and the pressures at the two ends are $\frac{2W}{\sqrt{7}}$ and $\frac{W}{\sqrt{7}}$.

(b) Two uniform rods AB, AC of equal lengths and each of weight W are pin-joined at A and are placed in a vertical plane wth the ends B and C resting on a smooth horizontal plane. Equilibrium is preserved by a stting whcih attaches C to the mid-point of AB. Show that the reaction of the rod at A and the tention in the string are both equal to $\frac{1}{4} W\sqrt{1 + 9\cot^2 \alpha}$ and each is inclined at an angle $\tan^{-1}\left(\frac{1}{3}\tan\alpha\right)$ to the horizontal, where α is the inclination of either rod to the horizontal. 3,2½

7. (a) A uniform ladder of length l rest on a rough horizontal ground with its upper end projecting slightly over a smooth horizontal rail at a height α. If the ladder is about to slip and λ is the

angle of friction with the groun, prove that $\tan\lambda = \frac{\alpha\sqrt{\ell^2 - \alpha^2}}{\ell^2 + \alpha^2}$.

(b) A heavy uniform rod of length 2a rests party wihin and partly outside a fixed smooth hemispherical bowl of radius r. The tim of the bowl is horizontal and one point of the rod is in contact with the rim, if 0 is the inclination of the rod with th horizontal, prove that $2r\cos 2\theta = \alpha \cos \theta$. Also prove that the greatest inclination of the rod that can thus rest is $\sin^{-1}\left(\frac{1}{\sqrt{3}}\right)$.

3,2½

8. (a) Find the centre of gravity of a solid hemisphere.
(b) A uniform rod rests in the limiting equilibrium with in a rough hollow sphere. If the rod subtends a right angle at the centre of the sphere, show that its inclination to the horizontal is twice the angle of friction. 3,2½

Section - C

9. (i) Particle of weight 3, 4, 5 and 6 gm are placed at corners A, B, c and D respectively of a rectangel ABCD. If AB = 6 cm and BC = 12 cm, find the perpendicualr distances of C.G. from AB and BC.
(ii) A mass of 40 kg. is supported by two fine light strings inclined at angles of 30° and 60° to the vertical. Determine the tensions of the strings.
(iii) State $\lambda - \mu$ Theorem.
(iv) ABC is triangle and G its centroid. A force R acts along AG, resolve R into two forces parallel to it and acting at the point B and C.
(v) The resultant of two coplanar forces is 36 newton and is inclined at an angle of 60° to the X–axis. If one of the forces is 24 newton along the X–axis, find the other force.
(vi) Explain three kinds of sliding friction.
(vii) Two forces equal in magnitude, act on a particle. Find the angle between the forces when the square of their resultant is equal to three times their product. 7×2=14