STATISTICAL METHODS - I

Time Allowed : Three Hours **Maximum Marks : 100**

Note : The candidates are required to attempt ONE question each from Sections A, B, C and D carrying 20 marks each and the entire Section E consisting of 10 short answer type questions carrying 2 marks each.

SECTION – A

1. Define the concept of statistic, parameter and explain the problem of point estimation. What are the criteria which should be satisfied by a good estimator ? 20
2. Explain the concept of Interval Estimation. Construct a 95% confidence interval for the difference between two population proportions, assuming both the populations being normal. 20

SECTION - B

3. Let $X_1, X_2, \ldots, X_n$ be a random sample from a $N(\mu, \sigma^2)$ distribution. Also let $\bar{X} = \frac{1}{n}\sum_{i=1}^{n} X_i$ and $s^2 = \frac{1}{n}\sum (x_i - \bar{x})^2$. Write down the probability density functions of $\bar{x}$, s^2 and their joint distribution. 20

4. Let X_i $(i = 1, 2, \ldots, n)$ be independent normal variates with means μ_i and variances σ_i^2 and let $z = \sum_{i=1}^{n}\left(\frac{X_i - \mu_i}{\sigma_i}\right)^2$, then derive the distribution of z. Also give the name of this distribution. 20

SECTION - C

5. Explain the following in the context of testing of hypotheses :
(i) critical value of a test, (ii) level of significance of a test, and (iii) one-tailed and two-tailed tests. 20

6. A random sample of 25 pairs of observation from a bi-variate normal distribution gives $r = 0.2$. Can the sample be regarded from the population having uncorrelated variables ? 20

SECTION - D

7. Discuss the chi-square test of goodness of fit. How will you determine the degrees of freedom of the chi-square distribution when some of the parameters of the theoretical distribution being fitted are estimated from the data itself ? 20

8. A machine producing screws turned 20 defective screws in the first run of 500 screws. After the first run the machine was overhauled and it turned 15 defective screws in the second run of 500 screws. Does this observation suggests that the machine has improved ?

Use a test of level of significance $\alpha = 0.05$. (Given $\int_{1.645}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz = 0.05$) 20

SECTION - E

9. Attempt all parts and do as directed :
(a) Give an example of a consistent estimator that is not unbiased.
(b) What is the S.E. of sample proportion in random sampling from an infinite population ?
(c) Define the Student's t-statistic.
(d) If X_1, X_2 are independent Poisson variates, what is the distribution of $X_1 + X_2$?
(e) Specify the moment generating function of a binomial distribution.
(f) Specify the parameters of a chi-square distribution.
(g) What is a null hypothesis ?
(h) What do you mean by p-value of a test statistic ?
(i) What do you understand by confidence coefficient ?
(j) Under what assumption the expected frequencies for a chi-square test of independence of two attributes are determined ? $2 \times 10 = 20$