3E1626

Roll No.

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B. Tech. (Sem. III) (Main/Back) Examination, December - 2017
Civil Engg.
3CE6A Advanced Engg. Mathematics

Time : 3 Hours

Maximum Marks : 80
Min. Passing Marks : 26

*Attempt any **five questions**, selecting **one question** from **each unit**. All Questions carry **equal** marks. Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly. Units of quantities used / calculated must be stated clearly.*

Use of following supporting material is permitted during examination. (Mentioned in form No. 205)

1. Nil 2. Nil

UNIT - I

1 (a) Find the Fourier series to represent $x - x^2$ from $x = -\pi$ to $x = \pi$. Hence show that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots\ldots = \frac{\pi^2}{12}$$

8

(b) Find the Z-transform of

(i) $\sin\theta$ 4

(ii) $c^n \cos n\theta$ 4

OR

1 [P.T.O.

1 (a) Find the inverse z-transform of $\frac{z^3 - 20z}{(z-2)^3(z-4)}$. 8

(b) Analyse Harmonically the data given below and express y in Fourier series up to third harmonic : 8

x :	0	$\pi/3$	$2\pi/3$	π	$4\pi/3$	$5\pi/3$	2π
y :	1.0	1.4	1.9	1.7	1.5	1.2	1.0

UNIT - II

2 (a) Find Laplace transform of $f(t) = \frac{1 - \cos 2t}{t}$. Also calculate the integral $\int_0^\infty e^{-t} \frac{\sin^2 t}{t} dt$. 8

(b) Solve $(D^2 - D - 2)x = 20 \sin 2t, x(0) = -1, x'(0) = 2$. 8

OR

2 (a) Apply convolution theorem and evaluate inverse Laplace transform of $\frac{s^2}{(s^2 + a^2)(s^2 + b^2)}$ 8

(b) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ where $0 < x < 1, t > 0$ together with the conditions $u(x, 0) = 3 \sin 2\pi x$, $u(0, t) = 0, u(1, t) = 0$. 8

UNIT - III

3 (a) Find the Fourier transform of $f(x)$, where

$$f(x) \begin{cases} 1 - x^2, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$$

and hence evaluate $\int_0^\infty \left(\frac{x \cos x - \sin x}{x^3} \right) \cos\left(\frac{x}{2}\right) dx$. 8

P.T.O.

(b) Solve $f(x)$ the integral equation

$$\int_0^\infty f(x)\cos(sx)\,dx = \begin{cases} 1-s & , \ 0 \le s \le 1 \\ 0 & , \ s > 1 \end{cases}$$

Hence prove that $\int_0^\infty \frac{\sin^2 t}{t^2}\,dt = \pi/2$ 8

OR

3 (a) Find Fourier cosine transform of $\frac{1}{1+x^2}$. 8

(b) Solve $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$, $x > 0, t > 0$ subject to the conditions :

(i) $u(0, t) = 0$

(ii) $u = \begin{cases} 1, & 0 < x < 1 \\ 0, & x \ge 1 \end{cases}$, when $t = 0$

(iii) $u(x, t)$ is bounded. 8

UNIT - IV

4 (a) Prove that

(i) $\Delta = \frac{1}{2}\delta^2 + \delta\sqrt{1+\frac{1}{4}\delta^2}$ 4

(ii) $e^x = \left(\frac{\Delta^2}{E}\right)e^x \cdot \frac{Ee^x}{\Delta^2 e^x}, h = 1$ 4

(b) Using Newton's forward interpolation formula, find y at $x = 8$ from the following table :

x :	0	5	10	15	20	25
y :	7	11	14	18	24	32

8

OR

P.T.O.

4 (a) Using Lagrange's interpolation formula, find the value of y at $x = 10$ from the following table :

x :	5	6	9	11
y :	12	13	14	16

8

(b) Evaluate $\int_0^1 \frac{1}{1+x^2} dx$ by using :

(i) Simpson's $\frac{1}{3}$ rule

(ii) Simpson's $\frac{3}{8}$ rule.

Also find approximate value of π. 8

UNIT - V

5 (a) Apply Picard method to find solution of differential equation $\frac{dy}{dx} = 1 + y^2$ and $y(0) = 0$ up to fourth order.

8

(b) Given $\frac{dy}{dx} = x^2(1+y)$ and $y(1) = 1$ then calculate $y(1.4)$ by Milne's PC method.

8

OR

5 (a) Given $\frac{dy}{dx} = x^2 - y$, $y(0) = 1$. Find $y(0.2)$ by Runge-Kutta Fourth order method.

8

(b) Given that $\frac{dy}{dx} = x + y^2$ and $y = 1$ at $x = 0$. Find an approximate value of y at $x = 0.4$ by modified Euler's method.

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