Roll No.

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3E1646

B. Tech III Sem. (Main/Back) Exam. Jan. 2016 **Electrical & Electronics Engineering** 3EX6A Advanced Engineering Mathematics - I Common to EE, EX

Time: 3 Hours

Maximum Marks: 80

Min. Passing Marks: 24

Instructions to Candidates:

Attempt any five questions, selecting one question from each unit. All questions carry equal marks. Schematic diagrams must be shown wherever necessary. Any data you feel missing suitably be assumed and stated clearly.

Units of quantities used/calculated must be stated clearly.

Use of following supporting material is permitted during examination. (Mentioned in form No. 205)

1. NIL

## UNIT-I

Q.1 (a) Find Laplace transform of t2H (t - 3)

[4]

Find inverse Laplace transform of  $\tan^{-1}\frac{2}{c^2}$ . (b)

- [4]
- Solve the following differential equation using Laplace transform technique: (c)

$$(D^4-1) y=1, y(0) = y'(0) = y''(0) = y'''(0) = 0$$

[8]

Q.1 (a) Find Laplace transform of  $\frac{e^{-t}\sin t}{t}$ .

[4]

(b) Find inverse Laplace transform of  $\frac{se^{-2s}}{s^2-1}$ .

[4]

(c) Solve the following partial differential equation using Laplace transform technique:

$$\frac{\partial u}{\partial t} = 2 \frac{\partial^2 u}{\partial x^2}$$

subject to the constraints

$$u(0, t) = u(5, t) = 0$$
;  $u(x, 0) 10 \text{ sm } 4\pi x$ .

# **UNIT-II**

Q.2 (a) Express the function -

$$f(x) = \begin{cases} \frac{2}{\pi} \sin x & ; \ 0 \le x \le \pi \\ 0 & ; \quad x > \pi \end{cases}$$

in integral form with the help of Fourier sine transform and hence evaluate

$$\int_0^\infty \frac{\sin(\pi \lambda)\sin(x \lambda)}{(1-\lambda^2)} d\lambda$$

[6+2=8]

(b) Find discrete Fourier transform of the sequence  $\{d_q\} = \{1, 2, 3, 4\}$ . [8]

OR

Q.2 (a) Prove that 
$$-\int_0^\infty \frac{\cos x \, \lambda}{\left(1 - \lambda^2\right)} \, d\lambda = \frac{\pi e^{-x}}{2}, \quad x \ge 0$$

[8]

(b) Solve the following partial differential equation

$$\frac{\partial \mathbf{u}}{\partial \mathbf{t}} = \frac{\partial^2 \mathbf{u}}{\partial \mathbf{x}^2} \quad (\mathbf{x} > 0, \mathbf{t} > 0)$$

subject to the constraints

$$u = 0$$
, when  $x = 0$ ,  $t > 0$ 

$$u = \begin{cases} 1 & ; & 0 < x < 1 \\ 0 & x \ge 1 \end{cases}, \text{ when } t = 0$$

[8]

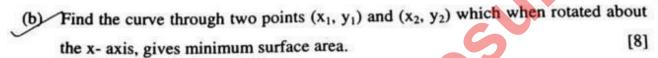
## **UNIT-III**

Q.3 (a) Find Fourier series to represent the function -

[8]

$$f(x) = \begin{cases} 0 & ; -\pi < x < 0 \\ \frac{\pi x}{4} & ; 0 < x < \pi \end{cases}$$

Hence deduce that  $\frac{\pi^2}{8} = \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots$ 



#### OR

- Q.3 (a) Find half range cosine series for the function  $f(x) = x(\pi x)$ ;  $x \in (0, \pi)$  [8]
  - (b) Find the shape of the plane curve of fixed length 1 so that it encloses maximum area. [8]

## UNIT-IV

- Q.4 (a) Show that the function  $f(z) = \sqrt{|x|y|}$  is not regular at origin, although Cauchy Riemann equations are satisfied. [8]
  - (b) Find the image of the infinite step  $\frac{1}{4} \le y \le \frac{1}{2}$  under the transformation  $w = \frac{1}{z}$ .

    Also show the regions graphically. [6+2=8]

### <u>or</u>

- Q.4 (a) Find the bilinear transformation that maps the points 0, -i, -1 in z-plane onto the points w = i, 1, 0. What are the invariant points of this transformation? [6+2=8]
  - (b) Using Cauchy's integral formula, evaluate -

$$\int \frac{z \, dz}{(z-2) (z-3)^2} \text{ where C is the circle } |z-3| = \frac{1}{2}.$$
 [8]

## UNIT-V

Q.5 (a) Find Laurent's series for the function  $f(z) = \frac{1}{z(1-z)}$  in the region -

(i) 
$$|z + 1| < 1$$

[21/2]

(ii) 
$$|<|z+1|<2$$

[21/2]

(iii) 
$$|z+1| > 2$$

[3]

(b) Find the residue of f (z) at each of its singularity, where f (z) is -

(i) 
$$\frac{1+e^{Z}}{\sin z + z\cos z}$$

[4]

(ii) 
$$\frac{e^{z}}{z(z+1)^{2}}$$

[4]

#### OR

Q.5 (a) Using Cauchy's residue theorem, evaluate the integral  $\int_c \frac{z-3}{z^2+2z+5} dz$ ,

where C is the curve 
$$|z+1-i|=2$$
.

[8]

(b) Show that -

$$\int_{-\infty}^{\infty} \frac{x^2 - x + 2}{x^4 + 10x^2 + 9} \, dx = \frac{5\pi}{12}$$

[8]

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