ALGEBRA AND MATHEMATICAL METHODS–III (i)

Time Allowed : Three Hours] [Maximum Marks : 100

Note : The candidates are required to attempt *one* question each from Section A, B, C and D carrying 20 marks each and the entire Section E consisting of 8 short answer type questions carrying 2½ marks each. Log tables, simple calculator only but not scientific calculator may be allowed.

Section : A

1. (i) A non empty subject H of a group G is a subgroup if and only if $a ; b \in H \Rightarrow ab \in H$ and $a \in H \Rightarrow a^{-1} \in H$. 8

   (ii) Let G and G' be two groups and $f : G \to G'$ be homomorphism of G onto G'. If H is the kernel of f, then $G/H \cong G$. 12

2. (i) Every quotient group of a cyclic group is cyclic. 6

   (ii) Prove that if for a group G, $f : G \to G$ is given by $f(x) = x^3$, $x \in G$ is an isomorphism, then G is abelian. 8

   (iii) Any finite cyclic group of order n is isomorphic to the quotient group $Z/\langle n \rangle$. 16

Section : B

3. (i) Intersection of a family of subrings of a ring R is subring or R. 6

   (ii) State and prove Fundamental Theorem of Ring Homomorphism.

   (iii) Show that the mapping $f : C \to M_2(R)$ defined by

   $$f(a + ib) = \begin{bmatrix} a & b \\ -b & a \end{bmatrix}$$ is a homomorphism of rings.

   Find kernel f. Is an isomorphism of C onto $M_2(R)$? 7

4. (i) Let R and S be two rings. A homomorphism $f : R \to S$ is one-one i.e. injective if and only if kernel $f = \{0\}$. 6

(ii) Let I and J be two ideals of a ring R so that $I \subseteq J$, then $\frac{(R/I)}{(J/I)} \cong R/J$. 7

(iii) Find the Greatest Common Divisor (gcd) of 11 + 7i and 18 – i in Z [i]. 7

Section : C

5. (i) Prove that order of convergence of Newton Raphson Method for approximation to root is 2. 10

(ii) Find a real root of the equation $2x = \cos x + 3$ correct to three decimal place using fixed point iteration method or method of simple iteration of the method of successive approximation. 10

6. (i) Explain Gauss Jordan Method for solving three linear system of equations with three variables. 10

(ii) Solve the following system of equations by Gauss-Seidel Method:
$5x_1 + 2x_2 + x_3 = 12$, $x_1 + 4x_2 + 2x_3 = 15$, $x_1 + 2x_2 + 5x_3 = 20$ upto five iterations. 10

Section : D

7. (i) Fit a polynomial of degree 3 using Newton forward difference formula which takes the following values :

x	:	3	4	5	6
y	:	6	24	60	120

8

(ii) Describe Bessel's central difference formula for interpolation near the middle of the tabulated values. 12

8. (i) Derive Everett's central difference formula by taking central ordinate as y_0 corresponding to $x = x_0$. 12

(ii) Find the polynomial which takes the following values using Newton's Backward interpolation formula as :

x	0.1	0.2	0.3	0.4	0.5
f (x)	1.40	1.56	1.76	2.0	2.28

Hence or otherwise obtain f (0.35). 8

Section : E

9. Do as directed :

(i) Every cyclic group is belian.

(ii) Let Z and E be the two groups of all integers and even integers under addition. Then show that the mapping $f : Z \to E$ defined by $f(x) = 2x$, $\forall\ x \in Z$ is a homomorphism. Is it an isomorphism. 2½

(iii) Find the field of quotients of the integral domain Z. 2½

(iv) Define Euclidean Domain or Euclidean Ring. 2½

(v) Write advantages of Regula-Falsi Method. 2½

(vi) Briefly explain Jacobi's Method for solving set of three linear equation in three unknowns. 2½

(vii) Derive the relation between Bessel's formula and Everett's Formula. 2½

(viii) Given the set of tabulated points (1, –3), (3, 9), (4, 30), (6, 132). Obtain the value of y when x = 2 using Newton's divided difference formula. 2½

Symbols used

$\in$ means belor
$\subseteq$ means subset or equal to
$\forall$ means for all
C means set of complex numbers
Z means set of integers
$\Rightarrow$ means implies to
$M_2(R)$ means 2 × 2 order matrix containing elements as reals.