DISCRETE MATHEMATICS – I

Time Allowed : Three Hours] [Maximum Marks : 100

Note : The candidates are required to attempt *one* question each form Section A, B, C and D carrying 20 marks each and the entire Section E consistig of 10 short answer type questions carrying 2 marks each.

Section : A

1. (a) In a town of 10,000 families, it was found that 40% families buy newspaper A, 20% buy a newspaper B and 10% buy newspaper C. 5% families buy A and B, 3% buy B and C, and 4% buy A and C. If 2% families buy all the newspapers, find the number of families which buy :

   (i) A only (ii) None of A, B and C.

   (b) A box, A contain 2 white and 4 black balls. Another box B, contains 5 white and 7 black balls. A ball is transferred from box A to B. Then a ball is transferred from box B to box A. Find the probability that it will be white. 10+10

2. (a) Show that the intersection of two equivalence relations is further an equivalence relation.

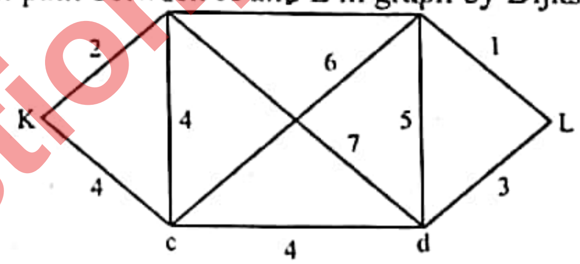
   (b) How many ways can a student do a 10 questions true-false exam if he or she can choose not to answer any number of questions ? 10+10

Section : B

3. (a) State and prove Euler's Formula for planar graph.

   (b) A graph G has 21 edges, 3 vertices of degree 4 and all other vertices are of degree 3. Find the number of vertices in G. 10+10

4. (a) Find the shortest path between K and L in graph by Dijkstra's Alogrithm.



   (b) What is preorder, postorder and inorder traversal of a binary tree ? 10+10

Section : C

5. (a) Solve $a_r - 4a_{r-1} + 3a_{r-2} = r^2$.

(b) Solve the recurrence relation $a_r + 5a_{r-1} + 6a_{r-2} = f(r)$, where $f(x)=\begin{cases}0, & r=0,1,5\\ 6, & \text{otherwise}\end{cases}$,

given that $a_0 = a_1 = 0$ 10+10

6\. (a) By finding generating function of the sequence $\{S(n)\}$, find the solution of the recurrence relation $S(n+2) - 7S(n+1) + 12S(n) = 0$. for $n \geq 0$, where $S(0) = 2$, $S(1) = 5$.

(b) Find the sequence having $\dfrac{1}{1-5z+6z^2}$ as a generating function. 10+10

Section : D

7\. (a) State and prove De Morgan's Laws for Boolean algebra.

(b) Show that the set Q_1 of al rational numbers other than 1 form a group under the operation $a * b = a + b - b$. 10+10

8\. (a) Prove that the complement of every element on a Boolean algebra B is unique.

(b) Show that the Boolean expressions $x \wedge (y \vee (y' \wedge (y \vee y')))$ and x are equivalent.

(c) Find the combinatorial circuit diagram for the Boolean expression $(x \wedge y') \vee (x \wedge z')$.

8+8+4

Section : E

9\. Do as directed :

(i) Prove by induction that for $n \geq 0$ and $a \neq 1$,

$$1 + a + a^2 + \ldots\ldots a^n = \frac{1-a^{n+1}}{1-a}$$

(ii) Define complement of an element in a Lattice.

(iii) Show that K_5 is not a planar graph.

(iv) Prove that every planar graph has at least one vertex of degree 5 or less than 5.

(vi) Solve the recurrence relation $S(K) - S(K-1)\, S(K-2) = 0$.

(vii) Find the numeric function corresponding to $\dfrac{3-5z}{(1-2z-3z^2)}$.

(viii) For a Boolean algebra prove that $a + (a * b) = a$.

(ix) Let $G = [V, E]$ be a graph having at least 11 vertices. Prove that G or its complement $\overline{G}$ is non-planar.

(x) Reduce the expression into simpler form by using the rules of Boolean algebra $AB + A\overline{C} + \overline{A}B\overline{C}(AB + C)$. 2×10=20