DISCRETE MATHEMATICS-I

Time : Three Hours] **[Maximum Marks : 100**

Note :- Attempt *one* question each from Sections A, B, C and D carrying 20 marks each. Section E consisting of 10 short answer type questions carrying 2 marks each is compulsory.

Section-A

I. (i) In a survey of 60 people, it was found that 25 read Newsweek magazine, 26 read Fortune, 9 read both Newsweek and Fortune, 11 read both Times and Newsweek, 8 read both Times and Fortune, 3 read all the three magazines. Find the number of people who read at least one of the three magazines and who read exactly one magazine.

(ii) The distinct equivalence classes of an equivalence relation on a set form a partition of that set.

II. (i) Find the number of different 8-letter arrangemnets that can be made from the letters of the word DAUGHTER so that all the vowels never occur together.

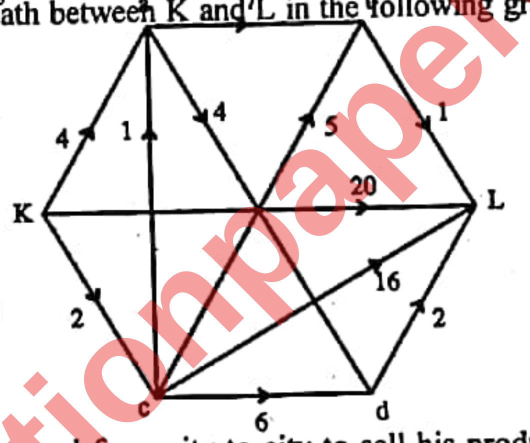
(ii) Consider a lettice L. Prove that the relation $a \leq b$ defined by either $a \wedge b = a$ or $a\, b = b$ is a partial ordering on lattice L. (10+10)

Section-B

III. (i) State and Prove Euler's Formula for planar graph.

(ii) Prove that an undirected graph G possesses an Eulerian circuit if and only if G is connected and degree of each vertex of G is even. (10+10)

IV. (i) Find the shortest path between K and L in the following graph by Dijkstra's Algorithm :



(ii) A salesman must travel from city to city to sell his product. The following table shows the distance (in km) between various cities :

		To City				
		A	B	C	D	E
From City	A	0	40	24	30	200
	B	40	0	25	300	30
	C	24	25	0	26	26
	D	30	300	26	0	40
	E	200	30	26	40	0

Use Assignment method to find the route of tour which will cover minimum total distance. (10+10)

Section-C

V. (i) Find the particular solution of the difference euqation

$$a_r - 4a_{r-1} + 4a_{r-2} = (r+1).\, 2^r$$

(ii) Solve $S_n - 4S_{n-1} + 3S_{n-2} = n^2$ (10+10)

VI. (i) Find the generating function from the recurrence relation given by

$S_n - 6s_{n-1} + 5S_{n-2} = 0$ Where $S_0 = 1$ and $S_1 = 1$

(ii) Define the Fibonacci sequence and find its generating function. (10+10)

Section-D

VII. (i) Consider an algebraic system (Q, *), where Q is the set of rational numbers and * is a binary operation defined by a * b = a + b - ab, for all a, b in Q. Determine whether (Q, *) is a group.

(ii) State and prove De Morgan's Laws for Boolean algebra. (10+10)

VIII. (i) Consider the lattice

$D_{60} = \{1, 2, 3, 4, 5, 6, 10, 12, 15, 20, 30, 60\}$.

(a) Draw the Hasse Diagram of D_{60}.

(b) Find all join-irreducible elements.

(c) Find all atoms.

(d) Find complements of 2 and 10, if they exist.

(e) Express each x as a join of irredundant join-ireeducible elements.

(ii) Find the circuit diagram of f(x, y, z) = xy' + x'z. (10+10)

Section-E

IX. Do as directed :

(a) Prove that A´ (BÈC)=(A´ B)È(A´ C)

(b) Define Partial ordered relation.

(c) Prove that the graph K_5 is not a planar graph.

(d) How many edges must a planar graph have if it defines 5 regions and has 6 vertices ?

(e) Discuss Konigsbeg's graph.

(f) If S(n) = n, n ≥ 0, then obtain its generating function.

(g) Prove that n(n + 1) (n + 2) is a multiple of 6, using mathematical induction.

(h) Define Complemented lattices.

(i) Prove that the complement of every element on a Boolean algebra B is unique.

(j) Show that the identity element in a group is unique. (2×10=20)