MATHEMATICAL METHODS - II

Note: Three Hours [Maximum Marks: 100]
Note: Attempt one question each from Sections A, B, C and D carrying 20 marks each. Section E is compulsory consisting of 10 short answer type questions carrying 2 marks each.

Section-A

Let f be bounded and integrable in [m. m] and manatonic in [m. m] and (0, m) (not

Let f be bounded and integrable in $[-\pi, \pi]$ and monotonic in $[-\alpha, 0)$ and $(0, \alpha]$, (not necessarily in the same sense), where δ is some positive number less than π . Prove that $\frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n = \frac{f(+0) + f(-0)}{\pi} \int_{0}^{\sin x} dx$ 1.

where a_n, n = 0, 1, 2, denote the Fourier's coefficients of f.
Obtain a Fourier cosine series of the periodic x², 0 ≤ x < π, of period π. 10, 10
(a) Find the Fourier series generated by the periodic function |x| of period 2π. Also compute the value at 0.
(b) if f is a bounded and integrable on [-π, π] and a_n, b_n are its Fourier coefficients, then

show that
$$\sum_{n=1}^{\infty} (a_n^2 + b_n^2)$$
 converges. Section-B

10, 10

Ш. (a)

Evaluate: (f(t)) when
$$f(t) = \begin{cases} 0, & 0 \le t < 1 \\ (t-1)^2, & t > 1 \end{cases}$$

Let f(t) be real and piecewise continuous function on each interval in $[0, \infty)$ and is of exponential order α . If L(f(t)) = F(s), then prove that **(b)**

$$L\left(\frac{f(t)}{t}\right) = \int_{0}^{\infty} F(s) ds$$

Prove that
$$\int_{0}^{\pi/2} \sqrt{\tan t} \ dt = \sqrt{\frac{\pi}{2}}$$

- **(b)** Apply convolution theorem to find inverse Laplace transform of
- V. State and prove convolution theorem for the Fourier Transforms.
- Evaluate $F_c^{-1}\left(\frac{1}{s^2+1}\right)$ Find finite Fourier cosine transform of f (t) = sin α t, t (0, π). VI.

$$f(x) = \begin{cases} 2, & 0 \le x \le \pi \\ 0, & x > \pi \end{cases}$$

Find Fourier sine integral of f(x) and evaluate.

$$\int_{0}^{\infty} \frac{1 - \cos s \pi}{s} \sin s \times ds$$

10, 10

Solve $(D^2 + 1) x = \sin 2t \sin t$, t > 0 when x = 1, Dx = 0 at t = 0. Find the solution of one-dimensional wave equation

Find the solution of one-dimensional wave equation
$$\frac{\partial^2 \mathbf{v}}{\partial \mathbf{v}} = \frac{\partial^2 \mathbf{v}}{\partial \mathbf{v}}$$

Find the solution of one-dimensional wave equation
$$\frac{\partial^2 y}{\partial t^2} = C^2 \frac{\partial^2 y}{\partial x^2}; x, t > 0$$
where $y(x, 0) = 0$, $y_t(x, 0) = 0$, $y(0, t) = f(t)$ and $y(x, t) \to 0$ as $x \to \infty$.

Solve
$$\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial x^2}$$
 with the following conditions
(i) $V_x(0, t) = 0$ for $t > 0$,
(ii) $V(x, 0) = \begin{cases} x, 0 \le x \le 1 \\ 0, x > 1 \end{cases}$
(iii) $V(x, t)$ is bounded.

10, 10

VIII. (a)

(i)
$$V_x(0, t) = 0$$
 for $t > 0$,

(ii)
$$V(x,0) = \begin{cases} x, 0 \le x \le 1 \\ 0, x > 1 \end{cases}$$

(b) Solve
$$\frac{dx}{dt} + x - y = 0$$
, $\frac{dy}{dt} + x + y = 0$, given $x(0) = 1$, $y(0) = 0$.

10, 10

(Compulsory Question)
Define Fourier transform and inverse Fourier transform of a function.
Find Fourier sine transform of

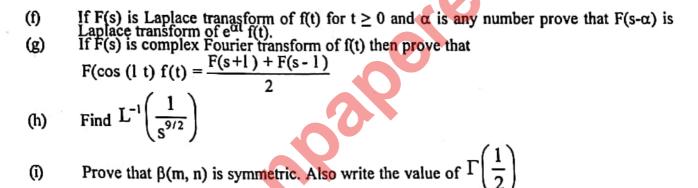
$$f(t) = \begin{cases} 2, & 0 \le t \le a \\ 0, & t > a \end{cases}$$

- Prove that for $t \ge 0$, $L(1) = \frac{1}{s}$, S > 1.
- Using First Shifting Theorem, prove that for

$$t \ge 0$$
, (L ($e^{\alpha t}$) = $\frac{1}{s-\alpha}$, $s > \alpha$.

(e)

Find Fourier transform of
$$f(t) = \begin{cases} \frac{\sqrt{2\pi}}{2a}, & -a < t < a \\ 0, & t < -a \text{ or } t > a \end{cases}$$



(f)

dx as Beta function. 0 $(10 \times 2 = 20)$