

MATHEMATICAL STATISTICS – I

Paper – III, Opt. II (i)

Time Allowed : Three Hours]

[Maximum Marks : 100

Note : Attempt one question each from Section A, B, C and D. Section E is compulsory. All questions carry equal marks.

Section : A

1. (a) An urn contains 5 white and 3 red balls and a second urn contains 4 white and 5 red balls. One of them is chosen at random and two balls are drawn from that urn. Find the change that they are of different colours. 6
- (b) A and B throw alternately with a pair of dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. If A begins, find his chances of winning. 7
- (c) Determine the expected number of families to have :
 - (i) at least one boy
 - (ii) no girl out of 800 families with 4 children each.
 Assume equal probabilities for boys and girls. 7
2. (a) The number of seed germinating out of 10 on damp filter paper of 80 sets of seeds is given. Fit a binomial distribution to be data :

x	0	1	2	3	4	5	6	7	8	9	10
y	6	20	28	12	8	6	0	0	0	0	0

7
- (b) The marks obtained in a certain examination are found to be normally distributed. If 12.5% of the candidates obtain 60% or more marks, 39% obtain less than 30 marks, find the mean number of marks obtained by candidates. (Given that for $x/\sigma = 1.15$, $A = 0.125$ and for $x/\sigma = 0.279$, $A = 0.61$). 7
- (c) If X be a binomially distributed random variable with $E(X) = 2$ and $\text{Var}(X) = 4/3$. Find the distribution of X. 6

Section : B

3. (a) There are three boxes containing respectively 1 white, 2 red, 3 black balls; 2 white, 3 red, 1 black balls and 3 white, 1 red, 2 black balls. A box is chosen at random and from it two balls are drawn at random. The balls are 1 red and 1 white. What is the probability that they come from second box? 10
- (b) If x and y are two random variables having joint density function:

$$f(x, y) = \begin{cases} \frac{1}{8}(6-x-y), & 0 < x < 2, 2 < y < 4 \\ 0 & \text{elsewhere} \end{cases}$$
 Then find $P((x < 1) / (y < 3))$ 10
4. (a) Let S_{200} be the number of heads that turn up in 200 tosses of a fair coin. Use the Central Limit Theorem to estimate $P(S_{200} = 90)$. 10
- (b) Let X be a continuous random variable with values uniformly distributed over the interval $[0, 20]$, then using law of large numbers calculate $p(|\bar{X} - 10| \geq 2)$. 10

Section : C

5. (a) A firm manufactures headache pills in two sizes A and B. Size A contains two grains of aspirin, five grains of bicarbonate and one grain of codeine. Size B contains one grain of aspirin, eight grains of bicarbonate and six grains of codeine. Studies have shown that it requires at least twelve grains of codeine for providing immediate effect. It is required to determine the least number of pills a patient should take to get immediate relief. Formulate this as an LP problem. 10
- (b) Use Simplex method to Maximize $Z = 6x + 7y$ subject to $7x + 6y \leq 42$, $5x + 9y \leq 45$, $x - y \leq 4$, $x, y \geq 0$. 10
6. (a) Solve the following LPP by Big M method :

$$\begin{aligned} \text{Minimize } Z &= 0.4x_1 + 0.5x_2 \\ \text{subject to } 0.3x_1 + 0.1x_2 &\leq 2.7, 0.5x_1 + 0.5x_2 = 6, \\ 0.6x_1 + 0.4x_2 &\geq 6, x_1, x_2 \geq 0. \end{aligned}$$
10
- (b) Solve the following LPP by two phase method :

$$\begin{aligned} \text{Minimize } Z &= 3x_1 + 2x_2 \\ \text{Subject to } 2x_1 + x_2 &\geq 10, -3x_1 + 2x_2 \leq 6, \\ x_1 + x_2 &\geq 6, x_1, x_2 \geq 0. \end{aligned}$$
10

Section : E

7. (a) Consider the problem Maximum $Z = -x_1 - 2x_2 - x_3$ subject to $x_1 + x_2 + 2x_3 \leq 12$, $x_1 + x_2 - x_3 \leq 1$, $x_1, x_2, x_3 \geq 0$. Using duality theory show that the optimal solution for the primal problem has $Z \leq 0$. 10

8. (b) Write a note on Vogel's approximation method. 10
 (a) Write a note on Dual Simplex method. 10
 (b) A company has factories at A, B and C which supply to warehouses at X, Y and Z locations. Weekly factory capacities are 200, 160 and 90 units respectively. Weekly warehouses requirements are 180, 120 and 150 units respectively. Unit shipping costs (in rupees) are as follows : 10

Factories	Warehouses			Supply
	X	Y	Z	
A	16	20	12	200
B	14	8	18	160
C	26	24	16	90
Demand	180	120	150	350

Determine the optimal distribution for this company to minimize shipping cost. 10
 Section : E

9. Attempt in brief :
 (a) A pair of dice is tossed twice. Find the probability of scoring 7 points twice.
 (b) Comment for a binomial distribution with mean 9 and standard deviation 20.
 (c) Write a note on correlation coefficient of bivariate random variables.
 (d) State Bayes theorem.
 (e) Write major steps in the solution of LPP by graphical method.
 (f) What is degeneracy in Simplex problems ?
 (g) Give optimality condition for transportation problem.
 (h) Is assignment problems special case of transportation problems ? Comment. 2½ marks each