

### Mathematical Statistics - III(ii)

**Time : Three Hours**

**Maximum Marks : 100**

**Note :** Attempt one question each from section A, B, C and D. Section E (Question No. IX) is compulsory. Use of scientific calculator is permitted.

#### Section-A

- I. (a) Five persons are chosen at random from a group containing 3 men, 2 women and 4 children. Find the chance that exactly two of them are children. (6)
- (b) A and B play alternately with a pair of coin, the first to get both heads wins the game. If A starts the first throw, find their chances of winning. (7)
- (c) Find the m.g.f. of a random variable  $X$  with density function as
- $$f(x) = \begin{cases} 2e^{-2x}, & x \geq 0 \\ 0, & x < 0 \end{cases} \quad (7)$$
- II. (a) A random variable  $X$  has probability density given by
- $$f(x) = \begin{cases} 3x^2, & 0 \leq x \leq 1 \\ 0, & \text{otherwise} \end{cases} \quad (6)$$
- Then find  $E(X^2)$
- (b) The probability of a bad reaction from a certain injection is 0.001. Determine the chance that out of 2000 individuals more than two will get a bad reaction. (7)
- (c) A random variable  $x$  is normally distributed with mean 12 and variance 4. Determine  $P(9.6 < x < 13.8)$  (7)
- (For  $x/s = 0.9$ ,  $A = 0.3159$ ;  $x/s = 1.2$ ,  $A = 0.3849$ )

#### Section-B

- III. (a) A box contains 3 blue and 2 red marbles, while another box contains 2 blue and 5 red

- marbles. A marble is drawn at random from one of the boxes and found to be red. What is the probability that it came from the second box? (10)
- (b) If  $X$  and  $Y$  are two random variables having joint probability function as
- $$f(x, y) = \begin{cases} c(2x + y), & 0 \leq x \leq 2, 0 \leq y \leq 3 \\ 0, & \text{elsewhere} \end{cases}$$
- IV. (a) Find conditional expectation of  $Y$  for  $X = 2$  (10)  
 Let  $S_{200}$  be the number of heads that turn up in 200 tosses of a fair coin. Use the Central Limit Theorem to estimate  $P(S_{200} = 90)$
- (b) Let  $X$  is normally distributed with mean  $m$  and variance  $s^2$ . Using Chebyshev's inequality, find an upper bound for the probability  $P(|X - \mu| \geq 3\sigma)$
- Section-C**
- V. (a) Using Iteration method, find a real root of  $2x - \log_{10} x = 7$  correct upto 4 decimals. (10)  
 (b) Using Newton-Raphson formula, find a real root of  $3x - \cos x - 1 = 0$  correct upto 4 decimal places. (10)
- VI. (a) Solve using Triangularization method  
 $x + y + z = 3$ ,  
 $2x - y + 3z = 16$ ,  
 $3x + y - z = -3$  (10)
- (b) Solve using Gauss Seidel method  
 $3x + 2y + z = 5$ ,  
 $2x + 5y + z = -3$ ,  
 $2x + y + 3z = 11$  (10)
- Section-D**
- VII. (a) Obtain the missing term in the following table :
- |          | 100    | 101     | 102    | 103     | 104     |
|----------|--------|---------|--------|---------|---------|
| $x$      | 2      | 2.00432 | —      | 2.01284 | 2.01703 |
| $f(x)$   | 0      | 5       | 10     | 15      | 20      |
| $\alpha$ | 1.5708 | 1.5738  | 1.5828 | 1.5981  | 1.6200  |
- (b) Compute  $f(9)$  using Bessel's formula from the table (10)
- VIII. (a) Calculate  $\log(41)$  from the data
- |          | 40      | 42      | 45      |
|----------|---------|---------|---------|
| $x$      | 40      | 42      | 45      |
| $\log x$ | 1.60206 | 1.52324 | 1.65321 |
| $x$      | 48      | 49      | 50      |
| $\log x$ | 1.68124 | 1.69019 | 1.69897 |
- (b) Estimate the value of  $\tan(0.12)$  from the table (10)
- |          | 0.10   | 0.15   | 0.20   | 0.25   | 0.30   |
|----------|--------|--------|--------|--------|--------|
| $x$      | 0.10   | 0.15   | 0.20   | 0.25   | 0.30   |
| $\tan x$ | 0.1003 | 0.1511 | 0.2027 | 0.2553 | 0.3093 |
- Section-E**  
 (Compulsory question)
- IX. Answer in brief :
- (a) A fair die is tossed twice. Find the probability of getting 4, 5 or 6 on the first toss and 1, 2 or 3 on the second toss.
- (b) Define Gamma random variable.
- (c) Explain the properties of Correlation coefficient.
- (d) Explain the concept of Uncertainty.
- (e) Write demerits of Newton-Raphson method.
- (f) Explain the concept of Complete pivoting.
- (g) Evaluate  $\Delta^3(1-x)(1-2x)(1-3x)$
- (h) Write Stirling formula (8×2½=20)