ANALYSIS – II

Paper - V : Semester-IV

Time Allowed : 3 Hours] [Maximum Marks : 36

Note : The candidates are required to attempt **two** questions each from Section A and B carrying 5.4 marks each and the entire Section C consisting of 7 short answer type questions carrying 2.06 marks each.

Section - A

1. (a) Show that the sequence $\{a_n\}$ where :

$a_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} \ldots\ldots \frac{1}{(n+1)!} + \frac{1}{n!}$ is convergent. 3

(b) Discuss the convergence of the series :

$\sum \left[\frac{1.3.5\ldots\ldots(2n-1)}{2.4.6\ldots\ldots(2n)}\right]^p$. where p is a real number. 2.4

2. (a) Prove that the sequence $\{x_n\}$ defined by :

$x_1 = \sqrt{7},\ x_{n+1} = \sqrt{7 + x_n}$ for $1 \geq 1$ converges to the positive square roots of $x^2 - x - 7 = 0.$ 3

(b) Using Cauchy's Integral test, discuss the convergence of the series :

$\sum_{n=1}^{\infty} \frac{1}{n^p}$. $p > 0$. 2.4

3. (a) State and prove Cauchy's First Theorem on limits. 3

(b) Discuss the convergence of the series :

$\sum_{n=1}^{\infty} \frac{1}{x^n + x^{-n}}$ for $x > 0$. 2.4

4. (a) Prove that sequence $\{a^n\}$ converges if $-1 < a \leq 1$. 3

(b) Show that for the series:

$$\frac{1}{2}+\frac{1}{3}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{2^3}+\frac{1}{3^3}+\frac{1}{2^4}+\frac{1}{3^4}+\ldots\ldots\ldots\ldots$$

Cauchy's Root Test indicates convergence, while D'Almert's Ratio Test is Inconclusive. 2.4

Section - B

5. (a) Test the sequence $f_n(x) = nx(1-x)^n$ for uniform convergence on the interval [0 1]. 3

(b) If $na_{n\to 0}$ and $f(x) = \sum a_n x^n \to s$ as $x \to 1-$, then $\sum a_n$ coverges to the sum s. 2.4

6. (a) Test the sequence $\frac{n^2x}{1+n^3x^2}$ for uniform convergence on the interval (0 1). 3

(b) Test the series: $(1-x)^2 + (1-x)^2 x + (1-x)^2 x^2 + \ldots\ldots\ldots\ldots$ for uniform convergence on the interval [0 1]. 2.4

7. (a) Show that:

$$\tan^{-1}x = x - \frac{x^3}{3} + \frac{x^5}{5} + \frac{x^7}{7} + \ldots\ldots\ldots \text{ for } -1 \leq x \leq 1.$$

Also deduce that $\frac{\pi}{4} = 1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots\ldots\ldots$ 3

(b) Show that

$$\int_0^x \log(1+x) = \sum_{n=1}^{\infty}(-1)^{n-1}\frac{x^{(n+1)}}{n(n+1)}, \text{ for } |x| < 1.$$ 2.4

8. (a) Examine for term by term integration the series:

$$\sum \frac{x}{(n+x^2)^2} \text{ for } 0 \leq x \leq 1.$$ 3

(b) Let $\sum_0^{\infty} a_n x^n$ be a power series with unit radius of convergence and let: $f(x) = \sum_0^{\infty} a_n x^n, -1 < x < 1$. If the series $\sum a_n$ converges, then prove that $\lim_{x\to 1-} f(x) = \sum a_n$. 2.4

Section - C

9. (a) Find the radius of convergences and interval of convergence of the series: $\sum_{n=1}^{\infty}(2n-1)x^n$.

(b) Show that the sequence $\{f_n(x)\}$ where: $f_n(x) = \frac{\sin nx}{\sqrt{n}}$ is uniformly convergent on $[0, 2\pi]$.

(c) Show that the series: $\sum \frac{a_n x^n}{1+x^{2n}}$ converges uniformly for all $x \in R$ if $\sum a_n$ is absolutely convergent.

(d) Prove that: $\lim_{n\to\infty} n\sqrt{n} = 1$.

(e) Show that $\left\{\frac{\log n}{n}\right\}$ is a null sequence. If $a_n = 1 + \frac{(-1)^n}{2n}$ find a positive integer m such that: $|a_n - 1| < \frac{1}{10^3}$ for all $n \geq m$.

(f) Show that $\lim_{n\to\infty} \sqrt{n^2+4n-1} = 2$.

(g) *Use cauchy's criteria to prove that divergence of the series :* $\sum \frac{1}{n^2}$.

7×2.06=14.4