DYNAMICS – VI (iii)

Semester-IV

Time Allowed : 3 Hours] [Maximum Marks : 36

Note : The candidates are required to attempt **two** questions each from Section A and B carrying 5½ marks each and the entire Section C consisting of 7 short answer type questions carrying 2 marks each.

Section - A

1 (a) O, A, B, C, D are five points in a straight line such that OA = AB = BC = CD. If a particle starts from rest with constant acceleration from O, prove that the times of describing AB, BC, CD are :

$(\sqrt{2}-1)\ (\sqrt{3}-\sqrt{2})\ (\sqrt{4}-\sqrt{3})$. 3

(b) A train weight M kg on the level is pulled by a force P kg wt. Against resistance R kg wt. Show that in developing velocity of V_1 from V_0m/sec the distance describe by train is $\frac{(V_1^2 - V_0^2)M}{2(P-R)g}$ meters. 2.5

2. (a) A ball is dropped from the top the tower ? meters hight and at the same moment another ball is projected upward from the bottom. They meet when the upper one has described $\frac{1}{n}$th of the total distance. Show that their speed when they meet are in ratio 2 : (n – 2) and initial velocity of lower ball is $\frac{1}{2}\sqrt{ng?}$. 3

(b) Two smooth inclined planes of inclination 30° and 60° respectively are placed back to back and a string, passing over a smooth pulley at the top, join massses of 0.3 kg and 0.5 kg lying on the planes. Find the acceleration of either mass, the tension in the string and the reaction of the planes. 2.5

3. (a) Two light strings are fastended to a particle of mass and other end of affixed point so the string are taut. Ther modulus of each is, the tension T and length a and b. Show that period of oscillation along the line of string is $2\pi\sqrt{\frac{map}{(T+\alpha)(a+b)}}$. 3

(b) A second's pendulum was too long on a given day by a quantity α, it was then over corrected so as to become too short by α during the next day. Prove that if L is the correct length, then the number of minutes gained in two days was $1080.\frac{\alpha^2}{l^2}$. 2.5

4. (a) A particle moves in straight line, starting from rest from a distance to a centre of attraction towards which the force per unit mass is $\frac{\mu}{x^3}$, where x is measured from the centre. Show that the time required to reach the centre is $\frac{c^2}{\sqrt{\mu}}$. 3

(b) The motion of particle in a straight line is given by the differential equation $x'' + n^2x + 0$ with initial condition $x = x_0$, $x' = v_0$ at t = o. Show that the motion is oscillatory and its amplitude is $\left[x_0^2 + \frac{v_0^2}{n^2}\right]^{1/2}$ and the initial phase is $\frac{\pi}{2}\tan^{-1}\left(\frac{v_0}{nx_0}\right)$. 2.5

Section - B

5. (a) Find latus rectum, vertices, the focus, the height of the directrix of the parabola traced out by a projectile. 3

(b) If t_1 and t_2 are two times of flight with which given range R on a horizontal plane can be reached by a particle with velocity u, prove that t_1 and t_2 satisfy the equation $g^2t^2 - 4u^2t^4 + 4R^2 = 0$. 2.5

6. (a) A spider hangs from ceilling room by a thread of elasticity equal to its weight. Show that it can climb to the ceiling with an expenditure of work equal to only three quarters of what be required if the thread were elastic. 3

(b) A uniform string of mass M and length 2a is placed symmetrically over a smooth peg and has particles of mass and m' attached its extremities. Show by the principle of enegy that when the string runss off the peg, its velocity is $\sqrt{\frac{M2(m-m')}{M+m+m'}}$.ga. 2.5

7. (a) ABC is a triangle right–angled at C; a particle P starts from A and moves along AC with uniform velocity u ; a second particle Q sarts from C at the same time instant and moves

along CB with uniform velocity v; show that the shortest distance between P and Q will be $\frac{v.AC}{\sqrt{u^2+v^2}}$ after a time $\frac{u.AC}{u^2+v^2}$. 3

(b) A bullet of mass m kg is fired into a fixed target of mass M kg and penetrates through a distance α meters. If the target was free to move, show that the distance penetrates would be $\frac{M\alpha}{M+m}$ meters and the K.K. lost would be $\frac{M}{M+m}$ of its initial distance. 2.5

8. (a) The mass of three spheres A, B, C are 7m, 7m, m; their co-efficient of restitutions is unity; their centres are in a straight line and C lies between A and B. Intially A and B are at rest and C is given a velocity along the line of centres towards A. Show that its strikes A twice and B once and final velocities of A, B, C are proportional to 21, 12, 1. 3

(b) A particle is projected with velocity $2\sqrt{ag}$ so that it just clear two walls of equal height α which are at a distance 2α from each other. Show that the latus-rectum of the path is 2α and that the time of passing between the walls is $2\sqrt{\frac{\alpha}{g}}$. 2.5

Section - C

9. (a) A particle is projected vertically upwards with velocity u, find maximum height attained.
(b) The position of the particle moving along the X-axis is given by $x = t^3 - 3t^2 + 10$. Determine distance covered by the particle in the time interval $t = 1$ to $t = 4$.
(c) Show that in recilinear motion with constant acceleration, the distance described in successive second from an A. P.
(d) A constant force acting on a mass of 5 kg. drags it 10 meters in 3 seconds. Find the force.
(e) Describe motion of a particle attached to an elastic string.
(f) If the greatest height attained by a projectile is $\frac{1}{4}$ of its range, find the angle of projection.
(g) Show that for a given velocity of projection, the maximum range down a pane of inclination α is greater than up the plane and are in the ratio $(1 + \sin\alpha) : (1 - \sin\alpha)$. 7×2=14