DYNAMICS-VI

(Semester-IV)

Time Allowed : Three Hours **Maximum Marks : 36**

Note : Attempt two questions each from Sections A and B carrying 5½ marks each, and the entire Section C consisting of 7 short answer type questions carrying 2 marks each.

SECTION-A

I. (a) A train weight M kg on the level is pulled by a force P kg wt. Against resistance R kg wt. Show that in developing velocity of V_1 from V_0 m/sec the distance described by train is $\frac{(v_1^2 - v_0^2)M}{2(P-R)g}$ metres. (2½)

(b) A string passes over a smooth fixed pulley and to one end mass m_1 is attached and to other end a smooth pulley over which passes another string with masses m_2 and m_3 at the ends. If the system is released from rest then show that m_1 will not move if $\frac{4}{m_1} = \frac{1}{m_2} + \frac{1}{m_3}$. (2½)

II. (a) A ball is dropped from the top of the tower ? metres high and at the same moment

another ball is projected upward from the bottom. They meet when the upper one has described $\frac{1}{n}$th of the total distance. Show that their speed when they meet are in ratio 2 : (n–2) and initial velocity of lower bail is $\frac{1}{2}\sqrt{ng}$? (2½)

(b) An aeroplane together with its load weights M kg and is falling with an acceleration of f m/sec², f being less than g. Show that if a part of the load equal to $\frac{2Mf}{f+g}$ kg . Be thrown out, the aeroplane will begin to rise with an acceleration of f m/sec². (2½)

III. (a) Find the escape velocity of a particle projected from the surface of earth. Where g = 9.8m/s² and R = 6370 km, R being the radius of earth. (2½)

(b) A point moving in a straight line with S.H.M. has velocities v_1 and v_2 when its distance from the centre are x_1 and x_2. Show that the period of motion is $2\pi\sqrt{\frac{x_1^2 - x_2^2}{v_2^2 - v_1^2}}$. (2½)

IV. (a) The motion of a particle in a straight line is given by the differential equation $x'' + n^2x = 0$ with initial condition $x = x_0$, $x' = v_0$ at $t = 0$. Show that the motion is osciallatory and its amplitude is $\left[x_0^2 + \frac{v_0^2}{n^2}\right]^{1/2}$ and the initial phase is $\frac{\pi}{2} - \tan^{-1}\left(\frac{v_0}{nx_0}\right)$. (2½)

(b) A simple pendulum has time period T. When the string is lengthened by a small fraction $\frac{1}{n}$ of its length, the period becomes T'. Show that $\frac{1}{n} - \frac{2(T - T')}{-T}$. (2½)

SECTION–B

V. (a) If t_1 and t_2 are two times of flight with which given range R on a horozontal plane can be reached by a particle with velocity u, prove that t_1 and t_2 satisfy the equation $g^2t^2 - 4u^2t^4 + 4R^2 = 0$. (2½)

(b) Find latus rectum, vertices, the focus, the height of the directrix of the parabola traced out by a projectivel. (2½)

VI. (a) A uniform string of mass M and length 2a is placed symmetrically over a smooth peg and has particles of masses m andm' attached to its extremities. Show by principle of energy that when the string runs off the peg, its velocity is $\sqrt{\frac{M + 2(m - m')}{M + m + m'}.ga}$ (2½)

(b) ABC is triangle right-angled at C; a particle P starts from A and moves along AC with uniform velocity u; a second particle Q stats from C at the same time instant and moves along CB with uniform velocity v; show that the shortest distance between P and Q will be $\frac{v.AC}{\sqrt{u^2 + v^2}}$ after a time $\frac{v.AC}{u^2 + v^2}$. (2½)

VII. (a) A bullet of mass m kg is fired into a fixed target of mass M kg and penetrates through a distance a metres. If the target was free to move, show that the distance penetrates would be $\frac{Ma}{M+m}$ metres and the K.E. lost would be $\frac{M}{M+m}$ of its initial distance. (2½)

(b) The mass of three spheres A, B, Care 7m, 7m, m; their co-efficient of restitution is unity, their centrs are in a straight line and C lies betwen A and B. Initially A and B are at rest and C is given a velocity along the line of centres towards A. Show that it strikes A twice and B once and final velocities of A,B,C are proportional to 21, 12, 1. (2½)

VIII. (a) A train of mass M pounds is ascending a smooth incline of 1 in n and when the

velocity of the train is v ft/sec, its acceleration is f ft/sec². Prove that the effective H.P of the engine is $\frac{Mv(nf+g)}{550ng}$. (2½)

(b) If v_1 and v_2 are the velocity at the ends of a focal chord of a projectile's path and u the horizontal component of velocity, show that $\frac{1}{v_1^2}+\frac{1}{v_2^2}=\frac{1}{u^2}$. (2½)

SECTION–C
(Compulsory Question)

IX. (a) Show that in rectlinear motion with constant acceleration, the distance described in successive seconds from an A.P.

(b) If the time of one complete oscillation of a simple pendulum is 20 seconds, find the length of the pendulum.

(c) A particle is projected vertically upwards with velocity u, find maximum height attained.

(d) A constant force acting on a mass of 5kg. Drags it 10metres in 3 seconds. Find the force.

(e) Compute the angular momentum of Neptune about the sun given that it moves in a circular orbit of radius 5×10^{12} meter. Mass of Neptune is 10^{26} kg and it completes one revolution in 165 years.

(f) Prove that the work done in stretching an elastic string is equal to the product of the extension and mean of the initial and final tension.

(g) Show that for a given velocity of projection, the maximum range down a plane of inclination α is greater than up the plane and are in the ratio $(1+\sin\alpha):(1-\sin\alpha)$.

(h) Show that the kinetic energy of a particle of mass m moving with a magnitude of velocity V is $\frac{1}{2}mV^2$. (8×2=16)