

NUMERICAL METHODS-IV

Semester-IV

Time Allowed : 3 Hours]

Note : The candidates are required to attempt two questions each from Section A and B carrying 5.4 marks each and the entire Section C consisting of 7 short answer type questions carrying 2.6 marks each.

Section - A

1. (a) Find the order of convergence of Newton-Rapson method
(b) Find a real root of the equation $x^2 - 4x - 9 = 0$ by Bisection method correct to three decimal places.
2. (a) Use Regular Falsi method to find a real root of $x^3 - 2x - 5 = 0$ Correct to four decimal places.
(b) Find the smallest root of the equation

$$1 - x + \frac{x^2}{(2!)^2} - \frac{x^3}{(3!)^2} + \frac{x^4}{(4!)^2} - \dots = 0$$

$3+2.4=5.4$

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3. (a) Using Triangularisation Method solve the equations :

$$\begin{aligned} 2x_1 + 3x_2 + 2x_3 &= 12 \\ 10x_1 + 3x_2 + 4x_3 &= 16 \\ 3x_1 + 6x_2 + x_3 &= -6 \end{aligned}$$
- (b) Find the order of convergence of Secant Method.
4. (a) Use Newton Raphson method to find a real root of the equation $x^3 + x^2 + 3x + 4 = 0$ correct to four decimal places.

$$3+2.4=5.4$$
- (b) Solve the following system of equations by Gauss Seidel method :

$$\begin{aligned} 5x + 2y + z &= 12 \\ x + 4y + 2z &= 15 \\ x + 2y + 5z &= 20 \end{aligned}$$
- $3+2.4=5.4$
5. (a) State and prove Lagrange's Interpolation formula
 Using Sterling formula, find y_3 , given that
 $y_0 = 512, y_1 = 439, y_2 = 346, y_4 = 243$
- (b) Find the polynomial (x) by using Lagrange's formula hence find $f(3)$ for :

$$\begin{array}{c|ccccc} x & 0 & 1 & 2 & 5 \\ \hline f(x) & 2 & 3 & 12 & 147 \end{array}$$
- (b) Use Everett's formula to evaluate $f(30)$
 If $f(20) = 2854, f(28) = 3162, f(36) = 7088, f(44) = 7984$
6. (a) State and Prove Bessel's Central Differences Formula
 (b) Use Newton Backward formula to estimate the number of students who obtained marks between 20 and 25 :
 Marks : 10 20 30 40 50
 No. of Students : 20 45 115 210 325
- $3+2.4=5.4$
7. (a) Using Newton's forward interpolation formula show that $\sum n^3 = \left\{ \frac{n(n+1)}{2} \right\}^2$
 (b) Find a polynomial using Bessel's interpolation from the table below :

$$\begin{array}{c|cccccc} X & 25 & 26 & 27 & 28 & 29 & 30 \\ \hline f(x) & 4.000 & 3.846 & 3.704 & 3.571 & 3.448 & 3.333 \end{array}$$
- $3+2.4=5.4$
8. (a) Evaluate $\Delta^n (e^x)$
 (b) Obtain the function whose first difference is $9x^2 + 11x + 5$
 (c) Using Lagrange's interpolation formula, express the function

$$\frac{x^2 + x + 3}{x^3 - 2x^2 - x + 2}$$
 as a sum of partial fractions
- (d) Solve the following system of equations by Gauss elimination method :

$$\begin{aligned} 2x + y + z &= 10 \\ 3x + 2y + 3z &= 18 \\ x + 4y + z &= 16 \end{aligned}$$
- (e) Mention the drawbacks of Newton Rapson method
- (f) Find the iteration formula for finding $\frac{1}{N}$ for some positive real number N using Newton Rapson method
 (g) Using iteration metho, find a root of the equation $x^3 + x^2 - 1 = 0$ correct to four decimal places.
- $7+2.06=14.4$