DISCRETE MATHEMATICS - II

Paper – IV
(Semester - VI)

Time Allowed : 3 Hours] [Maximum Marks : 36

Note : The candidates are required to attempt **two** questions each from Section A and B carrying 5.5 marks each and the entire Section C consisting of 10 short answer type questions carrying 1.4 marks each.

Section - A

1. (a) Give Big-O estimate for $f(n) = 3n \log n! + (n^2 + 3) \log n$.
   (b) Prove that $f(x) = 8x^3 + 5x^2 + 7$ is $\Omega(g(x))$, where $g(x) = x^3$.
2. (a) Solve $S_n + 5S_{n-1} + 6S_{n-2} = 3n^2 - 2n + 1$.
   (b) Find sequence whose generating function is $\frac{1}{1-z-z^2}$.
3. (a) Solve recurrence relation $S(n) - 4S(n-2) = 0$, $S(0) = 10$, $S(1) = 1$ for $n \geq 0$ using generating function.
   (b) For $S(n) = 3^n$ verify that $G(S * a, Z) = G(S, Z)\, G(a, Z)$.
4. (a) For the requrrence relation $a_n = 8a_{n-1} + 10n^{-1}$ with initial condition $a_0 = 1$. Find the generating function and the explicit formula for a_n.
   (b) Find generating function for the sequence of Fibonacci numbers.

Section - B

5. (a) Prove that set D_n of all positive divisors of an is a bounded distribution lattice.
   (b) Prove that for a bounded distributive lattice L, the complements are unique if they exist.
6. (a) Find the circuit $\left(x_1.\left(\left(x_1 - \bar{x}_3\right) + \left(\bar{x}_2.x_3\right)\right)\right) + \left(\bar{x}_1.x_2.x_3\right)$.

(b) Simplify the Boolean expression :

$$F(X,Y,Z)=(\bar{X}.Z)+(Y.Z)+(Y.\bar{Z})$$

and write in min. term normal form.

7. (a) Minimize the logic programme using K map :

$f(A, B, C, D) = \sum (0, 1, 2, 3, 5, 7, 8, 9, 10, 4)$.

(b) Reduce using Boolean rules $xy + xz + yz = xy + (x \oplus y)z$.

8. (a) Write the function $x \vee y'$ in the disjunction normal form in three variables x, y and z.

(b) Simplify the Boolean expression and make circuit diagram using NAND gate only.

$$F(A, B, C, D) = \bar{A}\bar{B}\bar{C}\bar{D} + \bar{A}\bar{B}C\bar{D} + A\bar{B}\bar{C}D + A\bar{B}CD + ABC\bar{D} + ABCD$$

Section - C

9. (a) Define ceiling function and characteristic function.

(b) If f be mod–11 function then find the value of f(–253).

(c) The numeric value of a defined as $a_r = \begin{cases} 2, & 0 \le 4 \le 3 \\ 2^{-r} + 5, & r \ge 4 \end{cases}$, find S^{-2} a.

(d) Determine C(5, 3) by recursive definition ob binomial coefficient.

(e) Write short note on recursion.

(f) Show that n, nth root of unity forms a group under multiplication.

(g) Prove that inverse of an element of group is unique.

(h) Draw operation table of G = {0, 1, 2, 3, 4, 5} under multiplication modulo 6.

(i) Define ring and sub ring.

(j) Prove that dual of distribution lattice is distributive. 10×1.4=14