

DISCRETE MATHEMATICS-II

Paper - IV
(Semester - VI)

Time Allowed : 3 Hours]

[Maximum Marks : 36

Note : The candidates are required to attempt two questions each from Section A and B carrying 5½ marks each and the entire Section C consisting of 7 short answer type questions carrying 2 marks each.

Section - A

1. Design an algorithm to select the largest and the second largest of n numbers. The basic operation is to compare the two numbers and determine the larger and the smaller of the two. What is the complexity of your algorithm? 5½
2. (a) If the solution of the recurrence relation $aS_n + bS_{n-1} + cS_{n-2} = 6$ is $3^n + 4^n + 2$, find a, b, c .
(b) Find generating function and sequence of recurrence relation $a_n + 2a_{n-1} = 0$ with $a_0 = 5$. 3+2½=5½
3. (a) By finding the generating function of sequence S_n , find the solution of recurrence relation $S_{n+2} - 7S_{n+1} + 12S_n = 0$ for $n \geq 0$, given $S_0 = 2, S_1 = 5$.
(b) If
$$a_r = \begin{cases} 0, & 0 \leq r \leq 2 \\ 2^{-r} + 5, & r \geq 3 \end{cases} \text{ and } \begin{cases} 3 - 2^r, & 0 \leq r \leq 1 \\ r + 2, & r \geq 2 \end{cases},$$
 find c, d , where $c_r = a_r + b_r, d_r = a_r b_r$. 3+2½=5½
4. (a) The Time complexity of algorithm A is $\Theta(r^2 \log n)$. Can we conclude that algorithm A is superior to algorithm B?
(b) Let a_n denotes the number of ways to seat 10 students in a row of r chairs so that no two students will occupy the adjacent seats. Determine the generating function of the discrete numeric $(a_0, a_1, a_2, \dots, a_r, \dots)$ 3+2½=5½

Section - B

5. (a) Prove that the set D_n of all positive divisors of n is a bounded distributive Lattice. 3+2½=5½
 (b) Show that Lattice with three or fewer elements is a chain.
 6. (a) Simplify the following Boolean expression and realize the logic diagram of the reduced expression with the help of NAND gate only :

$$\overline{ABCD} + \overline{ABC}\overline{D} + \overline{AB}\overline{CD} + \overline{AB}\overline{C}\overline{D} + \overline{A}\overline{BCD} +$$

$$\overline{A}\overline{BC}\overline{D} + \overline{A}\overline{B}\overline{CD} + \overline{A}\overline{B}\overline{C}\overline{D}.$$

$$3+2½=5½$$

- (b) Show that every chain is a distributive Lattice.
 7. (a) Let $E(x_1, x_2, x_3, x_4) = (x_1 \wedge x_2 \wedge \overline{x_3}) \vee (x_1 \wedge \overline{x_2} \wedge x_4) \wedge (x_2 \wedge \overline{x_3} \wedge \overline{x_4})$
 be a Boolean expression over the two valued Boolean algebra. Write E in both disjunctive and conjunctive normal forms.
 (b) Let (A, \wedge, \vee) be a boolean algebra. Show that (A, \oplus) is a commutative group, where \oplus is defined as $a \oplus b = (a \wedge \overline{b}) \vee (\overline{a} \wedge b)$. 3+2½=5½

8. (a) Give an example of a non-abelian group G and a normal subgroup H of G such that G/H is a abelian.
 (b) Define Complemented Lattice. Give an example of Complemented Lattice. 3+2½=5½

Section - C

9. (a) Find the particular solution of $a'' - 5a' + 6a = 3r^2$.
 (b) Show that $R = \{0, 1, 2, \dots, p-1\}$ under the operation addition modulo p and multiplication modulo p is a field iff p is prime.
 (c) Prove that POSET $P = \{2, 3, 4, 6\}$ under divisibility is not a Lattice.
 (d) Prove the validity of the following argument :
 If man is a Bachelor, he is unhappy. If a man is unhappy, he dies young.
 Therefore, bachelors dies young.
 (e) Prove that dual of distributive Lattice is Distributive.
 (f) Prove that every cyclic group is abelian, converse is not true, Justify.
 (g) Suppose $P(n) = a_0 + a_1n + a_2n^2 + \dots + a_m n^m$.
 Suppose $\deg P(n)$ is m . Prove that $P(n) = O(n^m)$. 7×2=14