MATHEMATICAL METHODS- II (iii)

Paper - IV (Semester - VI)

Time Allowed: 3 Hours]

Note: The candidates are required to attempt two questions each from Section A and B carrying 5½ marks each and the entire Section C consisting of 7 short answer type questions carrying 2

Section - A

1. (a) If $\overline{f}_c(p)$ is the Fourier cosine transform of the function f(x) which satisfy the Dirichlet conditions in every finite interval (0, a) and is such that

$$\int_{0}^{\infty} |f(x)| dx$$
exists, then
$$f(x) = \sqrt{\frac{2}{\pi}} \int_{0}^{\infty} \overline{f_{c}}(p) \cos px . dp$$

at every point of continuity of f(x). (b) Find the Fourier Transform of $f(x) = e^{-x^2/2}$.

3+21/2=51/2

2. (a) If $\overline{f_c}(p)$ is the Fourier cosine transform of the function f(x). Then Fourier consine transform of

$$\frac{1}{a}\overline{f_c}\left(\frac{p}{a}\right).$$

$$f(ax)$$
 is $\frac{1}{a}\overline{f_c}\left(\frac{p}{a}\right)$.

- Find the cosine transform of function of e-x and using the inversion formula recover the
- - State and Prove Modulation theorem. Let $f(x) = e^{-ax}$. Find $F_c(f(x))$ and using Parseval's Identity, prove that:

$$\int_{0}^{\infty} \frac{dx}{(a^{2} + x^{2})(b^{2} + x^{2})} = \frac{\pi}{2a(a + b)}.$$

- State and Prove Parsevel's Identity for Fourier transform.
 - Find Finite Fourier sine transform of $f(x) = x^3, 0 < x < a.$

- Section B
- (a) Using Laplace Transform, solve: $\frac{d^2y}{dt^2} + y = \sin 2t \sin t$, t > 0, where y(0) = 1, y'(0) = 0
 - Using Laplace Transform, solve : $\frac{d^2y}{dt^2} + (t-1)\frac{dt}{dt} y = 0$

where y(0) = 10,
$$\lim_{t \to \infty} y = 0$$
.

- Solve: $(D^2 + 2)x + Dy = 0$ $Dx (D^2 + 2)y = -1$, subject to x = y = Dx = Dy = 0 at t = 0.
 - Using Laplace transform, solve : $\frac{\partial^2 U}{\partial x^2} = \frac{1}{\alpha^2} \frac{\partial^2 U}{\partial t^2}$, x > 0, t > 0, where U(x, 0) = 0. $U_x(0, t) = c \sin(kt)$, $U_x(x, 0) = 0$ and U(x, t) is bounded as $x \to \infty$.
- A string is stretched between two points (0, 0) and (a, 0). If its is displaced into the curve 7.

$$y = k \sin\left(\frac{m\pi x}{a}\right)$$

5.

9.

and is released from rest in that position at time t = 0. Find displacement y(x, t) at any time t > 0and at any point $x \in (0, a)$. 51/2

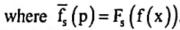
8.

using Finite Fourier transform, if V(0, t) = 0 and V(4, t) = 0 and V(x, 0) = rx, where x lies in (0, 4); t > 0. Section - C

Show that : $e^{-x} = \frac{2}{\pi} \int_{0}^{\infty} \frac{\cos px}{p^2 + 1} dp$, where $x \ge 0$,

using Fourier cosine integral transform.

- Explain Dirichlet's Conditions.
- If $F_c(f(x)) = \overline{f_c(p)}$ is the Fourier since Transform of f(x), then $F_c(f(ax)) = \frac{1}{a} \overline{f_c(\frac{p}{a})}$.
- Show that : $F_c(tf(t)) = \frac{d}{dp} \overline{f}_s(p)$,



- where $\overline{f_s}(p) = F_s(f(x))$. Find Finite Fourier sine and cosine transform of f(x) = 1 in $(0, \pi)$. (e)
- Solve, using Laplace Transform, $\frac{d^2y}{dt^2} + \pi^2y = 0$, (f)

where y(0) = 0, y(1) = 0. Write a short note on the choice of Infinite sine or cosine Fourier transform to solve Partial differential equation. $7 \times 2 = 14$ (g)