MATHEMATICAL METHODS- II (iii)

Paper - IV
(Semester - VI)

Time Allowed : 3 Hours] [Maximum Marks : 36

Note : The candidates are required to attempt two questions each from Section A and B carrying 5½ marks each and the entire Section C consisting of 7 short answer type questions carrying 2 marks each.

Section - A

1. (a) If $\overline{f}_c(p)$ is the Fourier cosine transform of the function f(x) which satisfy the Dirichlet conditions in every finite interval (0, a) and is such that

$$\int_0^\infty |f(x)|dx$$

exists, then

$$f(x)=\sqrt{\frac{2}{\pi}}\int_0^\infty \overline{f}_c(p)\cos px.dp$$

at every point of continuity of f(x).

(b) Find the Fourier Transform of $f(x)=e^{-x^2/2}$. 3+2½=5½

2. (a) If $\overline{f}_c(p)$ is the Fourier cosine transform of the function f(x). Then Fourier consine transform of

$\frac{1}{a}\overline{f}_c\left(\frac{p}{a}\right)$.

f(ax) is $\frac{1}{a}\overline{f}_c\left(\frac{p}{a}\right)$.

(b) Find the cosine transform of function of e^{-x} and using the inversion formula recover the original function. 3+2½=5½

3. (a) State and Prove Modulation theorem.
(b) Let $f(x) = e^{-ax}$. Find $F_c(f(x))$ and using Parseval's Identity, prove that :

$$\int_0^\infty \frac{dx}{(a^2+x^2)(b^2+x^2)} = \frac{\pi}{2a(a+b)}.$$ 3+2½=5½

4. (a) State and Prove Parsevel's Identity for Fourier transform.
(b) Find Finite Fourier sine transform of $f(x) = x^3$, $0 < x < a$. 3+2½=5½

Section - B

5. (a) Using Laplace Transform, solve : $\frac{d^2y}{dt^2} + y = \sin 2t \sin t,\ t > 0,$
where $y(0) = 1$, $y'(0) = 0$.

(b) Using Laplace Transform, solve : $\frac{d^2y}{dt^2} + (t-1)\frac{dt}{dt} - y = 0,$
where $y(0) = 10$, $\lim_{t\to\infty} y = 0$. 3+2½=5½

6. (a) Solve : $(D^2 + 2)x + Dy = 0$ $Dx - (D^2 + 2)y = -1$, subject to $x = y = Dx = Dy = 0$ at $t = 0$.

(b) Using Laplace transform, solve : $\frac{\partial^2 U}{\partial x^2} = \frac{1}{\alpha^2}\frac{\partial^2 U}{\partial t^2},\ x > 0, t > 0,$
where $U(x, 0) = 0$. $U_x(0, t) = c \sin(kt)$, $U_t(x, 0) = 0$ and $U(x, t)$ is bounded as $x \to \infty$. 3+2½=5½

7. A string is stretched between two points (0, 0) and (a, 0). If its is displaced into the curve

$$y = k\sin\left(\frac{m\pi x}{a}\right)$$

and is released from rest in that position at time $t = 0$. Find displacement $y(x, t)$ at any time $t > 0$ and at any point $x \in (0, a)$. 5½

8. Solve : $\frac{\partial V}{\partial t} = \frac{\partial^2 V}{\partial x^2}$,
using Finite Fourier transform, if $V(0, t) = 0$ and $V(4, t) = 0$ and $V(x, 0) = rx$, where x lies in (0, 4); $t > 0$. 5½

Section - C

9. (a) Show that : $e^{-x} = \frac{2}{\pi}\int_0^\infty \frac{\cos px}{p^2+1}dp$, where $x \geq 0$,
using Fourier cosine integral transform.
(b) Explain Dirichlet's Conditions.
(c) If $F_c(f(x)) = \overline{f}_c(p)$ is the Fourier since Transform of f(x), then $F_c(f(ax)) = \frac{1}{a}\overline{f}_c\left(\frac{p}{a}\right)$.
(d) Show that : $F_c(tf(t)) = \frac{d}{dp}\overline{f}_s(p)$,

where $\overline{f_s}(p) = F_s(f(x))$.

(e) Find Finite Fourier sine and cosine transform of $f(x) = 1$ in $(0, \pi)$.

(f) Solve, using Laplace Transform, $\dfrac{d^2y}{dt^2} + \pi^2 y = 0$,
where $y(0) = 0$, $y(1) = 0$.

(g) Write a short note on the choice of Infinite sine or cosine Fourier transform to solve Partial differential equation. $7 \times 2 = 14$