ALGEBRA

Paper-I

Semester-V

Time Allowed : 3 Hours] [Maximum Marks : 36

Note : The candidates are required to attempt two questions each from Sections A and B carrying 5½ marks eachand the entire Section C consisting of 7 short answer type questions carrying 2 marks each.

SECTION-A

1. (a) Show that if G is group in which $(ab)^n = a^n b^n$ for three consecutive integers n and a, b ∈ G, then G is abelian. 2½
   (b) State and prove Cayley's theorem. 3
2. (a) Prove that for any positive integer n, the set C_n of all n × n matrices over the complex numbers form an infinite abelian group under matrix addition as binary composition. 2½
   (b) Show that G={1,2,3,4} forms a cyclic group under multiplication modulo 5. 3
3. (a) Prove that Quotient group of a cyclic group is cyclic. 3
   (b) If H, K are normal subgroups of a group G and H⊂K, then K/H is a normal subgroup of G/H. 2½
4. (a) If $O(G) = p^2$, where p is a prime number, then G is abelian. 3
   (b) Prove that A_4 has no subgroup of order six. 2½

SECTION-B

5. (a) Prove that any field is an integral domain. 2½
   (b) If I and J be any two ideals of ring R, then IJ is an ideal of R moreover IJ ⊆ I ∩ J. 3
6. (a) An ideal P of a commutative ring R is a prime ideal iff R/P is an integral domain. 3
   (b) If I and J are two ideals of ring R and I ⊆ J, then J/I is ideal of R/I. 2½
7. (a) State and prove Fundamental theorem on Ring homomorphism. 3
   (b) Prove that every irreducible element of PID is a prime. 2½
8. (a) If R is an integral domain and a is irreducible element of R, then a is also irreducible element of R[x]. 2½
   (b) State and prove Gauss's lemma. 3

SECTION-C

9. Attempt the following questions : 7×2=14
   (i) Show that x^3 - [9] is irreducible over 31.
   (ii) Find the units of ring [i].
   (iii) Define Nilpotent ideal and give an example of Nilpotent ideal.
   (iv) Define principal ideal ring and UFD.
   (v) Give an example of integral domain which is to a field.
   (vi) Prove that intersection of two normal subgroup of G is a normal subgroup of G.
   (vii) State Lagrange's theorem. Converse of Lagrange's theorem is true for cyclic group or not.