B. TECH (SEM 4TH)

AM-201(2008 BATCH) : MATHEMATICS - III

PAPER ID : A0865

Time : 3 Hours *Maximum Marks : 60*

Instruction to candidates : (i) *Section-A is compulsory. Consisting of ten questions carrying two marks each*
*(ii) Section B contains Five questions carrying Five marks each and students has to attempt any **four** questions.*
(iii) Section C contains three questions carrying Ten marks each and students has to attempt any two questions.

SECTION-A

Q.1 (a) Let $f(x)$ be a periodic function of period 2π is the interval $(c,c+2\pi)$. and is represented by the following trigonometric series:

$$f(x) = \frac{a_o}{2} + \sum_{n=1}^{\infty} (a_n \text{Cos } nx + b_n \text{Sin } nx).$$

Write Enler formula to determine a_o, a_n, b_n.

(b) Find L{sin6t sin4t}.

(c) If $L\{f(t)\} = \bar{f}(s)$, then show that $L\{f(t-a)\, u(t-a)\} = e^{-as}\bar{f}(s)$.

(d) Solve : $p\tan x + q\tan y = \tan z$.

(e) Solve : $\frac{\partial^3 z}{\partial x^3} - 2\frac{\partial^3 z}{\partial x^2 \partial y} + \frac{\partial^3 z}{\partial x \partial y^2} = 0.$

(f) Evaluate $\int_0^{1+i} (x-y+ix^2)dz$ along the straight line from z=0 to z= 1+i.

(g) Evaluate $\oint \frac{e^z}{z-2} dz$, c:|z|=1.

(h) Prove that $J_0^1(x) = -J_1(x)$

(i) Show that $p_n(1)=1$.

(j) Define error function.

SECTION - B

Q.2 Find the laplace transform of $\frac{\sin 2t}{t}$.

Does the transform of $\frac{\cos 2t}{t}$ exist?

Q.3 (a) Prove that $\int_{-1}^{1} pn(x)dx=0$, if $n \neq 0$.

Q.4 Prove that e^z is analytic and find its first derivative.

Q.5 Use method of seperation of variables to solve :

$$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u, \quad u(x, 0)=6e^{-3x}.$$

Q.6 Solve $\frac{\partial^2 z}{\partial x^2} + 3\frac{\partial^3 z}{\partial x \partial y} + 2\frac{\partial^2 z}{\partial y^2} = x + y.$

SECTION - C

Q.7 Obtain Fourier series for the function defined below :

$$f(x) \begin{cases} \pi 2, & 0 \leq x \leq 1 \\ \pi(2-2), & 1 \leq x \leq 2 \end{cases}$$

Also draw the graph of $f(x)$.

Q.8 Apply calculas of residues to prove that $\int_{0}^{2\pi} \frac{d\theta}{1-2a\sin\theta+a^2} = \frac{2\pi}{1-a^2}$, $0 < a > 1$.

Q.9 An insulated rod of length 100cm has its ends A and B maintained at 0°C and 100°C respectively until steady stte conditions prevail. If end B is suddenly reduced to 0°C and maintained at 0°C, find the temperature at a distance x from A at time t.