ALGEBRA AND CALCULUS – II

Time Allowed : Three Hours] **[Maximum Marks : 100**

Note : The candidates are required to attempt *one* question each from Sections A, B, C and D carrying 20 marks each and the entire Section E consisting of 8 short answer types questions carrying 2½ marks each.

Section - A

1. (a) Find value of x for which $y = x^4 - 6x^3 + 12x^2 + 5x + 7$ is concave upwards or downwards. Also determine the point of inflexion.

(b) Find y_n if $y = \frac{1}{x^2 + a^2}$.

2. (a) Find radius of curvature at any point of the curve $x^{2/3} + y^{2/3} = a^{2/3}$.

(b) Trace the curve $x^3 + y^3 = 3axy$, $a \geq 0$.

Section - B

3. (a) Find Reduction formula for $\int \cot^n x \, dx$.

(b) Find length of Boundary of Region bounded by the curve $y = \frac{1}{2}x^2 + 1$ and the lines $y = x$, $x = 0$ and $x = 2$.

4. (a) State and prove DIRICHLET's TEST for Convergence at ∞.

(b) Show that $\left(m + \frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2m-1}}$

Section - C

5. (a) For the matrix $A = \begin{bmatrix} 1 & -3 & 3 \\ 3 & -5 & 3 \\ 6 & -6 & 4 \end{bmatrix}$; find the Invertible Matrix P, such that $P^{-1}AP$ isa diagonal matrix.

(b) Find the values of x suchthat the rank of matrix $\begin{bmatrix} 3x-8 & 3 & 3 \\ 3 & 3x-8 & 3 \\ 3 & 3 & 3x-8 \end{bmatrix}$ is ≤ 2. Also find rank for these values of x.

6. (a) State and prove Cayley–Hamilton theorem, using it find inverse of $\begin{bmatrix} 0 & 0 & 1 \\ 1 & 2 & 0 \\ 2 & -1 & 0 \end{bmatrix}$.

(b) Find the values of λ and μ so that system of equation

$2x - 3y + 5z = 12$

$3x + y + \lambda z = \mu$

$x - 7y + 8z = 17$

has :

(i) a unique solution (ii) Infinite solution

(iii) no solution.

Section - D

7. (a) If α and β be the roots of $x^2 - 2x + 4 = 0$, prove that

(i) $\alpha^n + \beta^n = 2^{n+1} \cos \frac{n\pi}{3}$ (ii) $\alpha^6 + \beta^6 = 128$.

(b) If $a^{\alpha + i\beta} = (x + iy)^{p+ib}$, then prove that

$\alpha = \frac{1}{2} p \log_a (x^2 + y^2) - q \tan^{-1} \frac{y}{x} \log_a e$ and

$\log_a (x^2 + y^2) = 2 \log \frac{p\alpha + q\beta}{p^2 + q^2}$

8. (a) Use CARDAN's method to solve :

$2x^3 - 7x^2 + 8x - 3 = 0$

(b) Change the equation $2x^5 + 3x^3 + 4x^2 - 1 = 0$, into in which coefficient of leading terms is unity and coefficient of other terms remains integers.

Section - E

9. Do as directed :

(a) Determine nature of double roots at origin for $x^2 (x - y) + y^2 = 0$.

(b) Show that the parabola $y^2 = 4ax$ has no asymptotes.

(c) Evaluate $\int_0^{\frac{\pi}{2}} (\sin x)^{2/3} (\cos x)^{-1/2} dx$.

(d) Test convergence of $\int_0^1 \frac{\sin(1/x)}{\sqrt{x}} dx$.

(e) Show that sin z is periodic of period 2π.

(f) Prove that if λ is Eigen value of A, ten λ^m is Eigen valueof $A^m \forall m \in N$.

(g) Discuss Nature of roots of $x^3 - 6x^2 + 9x - 2 = 0$, Locate them.

(h) Define Rank of Matrix and its one significance.