ANALYTIC GEOMETRY - III

Time Altowed: Three Hours

Maximum Marks: 100

Note: The candidates are required to attempt one question each from Sections A, B, C and D carrying 20 marks each and the entire Section E consisting of 8 short answer types questions carrying 21/2 marks each.

Section - A

Prove that the equation 1. (a) $x^2 + 2xy + y^2 - 2x - 1 = 0$

represents a parabola and find its focus, latus ractum and directrix.

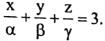
- Prove that the subnormal at any point of a parabola is constant and eaualto half of the latus (b)
- Prove that the locus of middle point of the normal chords of the parabola $y^2 = 4ax$ is (a) 2.
 - Prove that the semi latus ractum of a parabola is the harmonic mean between the segments (b) of a focal chord.

Section - B

- If the normal at the end point of a latus ractum of a ellipse passes through one extremity of (a) 3. the minor axes, show that the eccentricity of the curve is given by the equation $e^4 + e^2 - 1 = 0$
 - Find the pole of the line x 2y + 3 = 0 w.r.t. the ellipse (b) $3x^2 + 4y^2 = 12$
- Find the length of the intercept made by the hyperbola $\frac{x^2}{a^2} \frac{y^2}{b^2} = 1$ on the line y = mx + c. (a) 4.
 - If e and e' be the eccentricities of a hyperbola and of its conjugate hyperbola, then show (b)

Section - C

A plane meets the co-ordinate axes at A, B & C such that the centroid of \(\Delta ABC \) is the point (α, β, γ) . Show that the equation of the plane is:



- (b)
- Find the equation of the plane passing through the point (1, -2, 3) and and perpendicular to the perpendicular to the plane x y + 2z 3 = 0 and 3x + 2y z = 0. Find the length of perpendicular from the point (-1, 3, 2) on the plane x + 2y + 2z 3 = 0. 6. (a) Also find the co-ordinates of the foot of perpendicular.
 - Show that the S.D. between the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-4}{4} = \frac{z-5}{5}$ is (b) $\frac{1}{\sqrt{6}}$ and its equations are 11x + 2y + 7z + 6 = 0, 7x + y - 5z + 7 = 0.
- Section D 7. Find the equation of the sphere which passes through the points (1, 0, 0), (0, 1, 0), (0, 0, 1) (a)
 - and has its radius as small as possible. Find them centre and radius of the circle in which the sphere $x^2 + y^2 + z^2 + 2x 2y 4z 19 = 0$ is cut by the plane x + 2y + 2z + 7 = 0. Find the equation of the cone with vertex at origin and which passes through the curve (b)
- 8. (a) given by: $x^2 + y^2 + z^2 + x - 2y + 3z - 4 = 0$ and $x^2 + y^2 + z^2 + 2 - 3y + 4z - 5 = 0$. Prove that the equation:
 - (b) $7x^2 + 2y^2 + 2z^2 - 10zx + 10xy + 26x - 2y + 2z - 17 = 0$ represents a cone whose vertex is at the point (1, -2, 2) Section - E
- Dos as directed: 9.
 - Transform the equation $x^2 2xy + y^2 + x + y = 0$ to an equating in which term containing
 - Find the equation of tangent tot he parabola $y^2 = 6x$ at the point (8/3, 4)
 - Write down equation of director circle of the ellipse $16x^2 + 9y^2 = 144$

 - Define rectangular hyperbola and find its ecentricity. Find the equation of the straight line joining the points (-2, 1, 3) and (3, 1, -2).
 - Define cone and right circular cone.

 - Find the equation of the sphere concentric with $x^2 + y^2 + z^2 2x 4y 6z 11 = 0$ but of double its radius. Show that the planes x 2y + z = 0, x + y 2z = 3 and 3x 2y + z = 2 meet in a point.