ANALYTIC GEOMETRY - III

Time Allowed : Three Hours **Maximum Marks : 100**

Note : The candidates are required to attempt *one* question each from Sections A, B, C and D carrying 20 marks each and the entire Section E consisting of 8 short answer types questions carrying 2½ marks each.

Section - A

1. (a) Prove that the equation
   $x^2 + 2xy + y^2 - 2x - 1 = 0$
   represents a parabola and find its focus, latus ractum and directrix.
   (b) Prove that the subnormal at any point of a parabola is constant and eaualto half of the latus ractum.

2. (a) Prove that the locus of middle point of the normal chords of the parabola $y^2 = 4ax$ is
   $$\frac{y^2}{2a} + \frac{4a^3}{y^2} = x - 2a.$$
   (b) Prove that the semi latus ractum of a parabola is the harmonic mean between the segments of a focal chord.

Section - B

3. (a) If the normal at the end point of a latus ractum of a ellipse passes through one extremity of the minor axes, show that the eccentricity of the curve is given by the equation
   $e^4 + e^2 - 1 = 0$
   (b) Find the pole of the line $x - 2y + 3 = 0$ w.r.t. the ellipse
   $3x^2 + 4y^2 = 12$

4. (a) Find the length of the intercept made by the hyperbola $\frac{x^2}{a^2} - \frac{y2}{b^2} = 1$ on the line $y = mx + c$.
   (b) If e and e' be the eccentricities of a hyperbola and of its conjugate hyperbola, then show that :
   (i) $\frac{1}{e^2} + \frac{1}{e'^2} = 1$
   (ii) $e' = \frac{e}{\sqrt{e^2 - 1}}$

Section - C

5. (a) A plane meets the co-ordinate axes at A, B & C such that the centroid of ΔABC isthe point (α, β, γ). Show that the equation of the plane is :

$\frac{x}{\alpha}+\frac{y}{\beta}+\frac{z}{\gamma}=3.$

(b) Find the equationof the plane passing through the point (1, –2, 3) and and perpendicular to the perpendicular to the plane $x - y + 2z - 3 = 0$ and $3x + 2y - z = 0$.

6. (a) Find the length of perpendicular from the point (–1, 3, 2) on the plane $x + 2y + 2z - 3 = 0$. Also find the co-ordinates of the foot of perpendicular.

(b) Show that the S.D. between the lines $\frac{x-1}{2}=\frac{y-2}{3}=\frac{z-3}{4}$ and $\frac{x-2}{3}=\frac{y-4}{4}=\frac{z-5}{5}$ is $\frac{1}{\sqrt{6}}$ andits equationsare

$11x + 2y + 7z + 6 = 0,\ 7x + y - 5z + 7 = 0.$

Section - D

7. (a) Find the equationof the sphere which passes through the points (1, 0, 0), (0, 1, 0), (0, 0,1) and has its radius as small as possible.

(b) Find thenm centre and radius of the circle in which the sphere $x^2 + y^2 + z^2 + 2x - 2y - 4z - 19 = 0$ is cut by the plane $x + 2y + 2z + 7 = 0$.

8. (a) Find the equation of the cone with vertex at origin and which passes through the curve given by :

$x^2 + y^2 + z^2 + x - 2y + 3z - 4 = 0$ and $x^2 + y^2 + z^2 + 2 - 3y + 4z - 5 = 0$.

(b) Prove that the equation :

$7x^2 + 2y^2 + 2z^2 - 10zx + 10xy + 26x - 2y + 2z - 17 = 0$

represents a cone whose vertex is at the point (1, –2, 2)

Section - E

9. Dos as directed :

(a) Transform the equation $x^2 - 2xy + y^2 + x + y = 0$ to an equatino in which term containing xy is missing.

(b) Find the equation of tangent tot he parabola $y^2 = 6x$ at the point (8/3, 4)

(c) Write down equation of director circle of the ellipse $16x^2 + 9y^2 = 144$

(d) Define rectangular hyperbola and find its ecentricity.

(e) Find the equation of the straight line joining the points (–2, 1, 3) and (3, 1, –2).

(f) Define cone and right circular cone.

(g) Find the equation of the sphere concentric with $x^2 + y^2 + z^2 - 2x - 4y - 6z - 11 = 0$ but of double its radius.

(h) Show that the planes $x - 2y + z = 0$, $x + y - 2z = 3$ and $3x - 2y + z = 2$ meet in a point.