2X7-14

ANALYTIC GEOMETRY

Paper - VI : Semester-II

Time Allowed : 3 Hours] [Maximum Marks : 36

Note : The candidates are required to attempt **two** questions each from Section A and B carrying 5½ marks each and the entire Section C consisting of 7 short answer type questions carrying 2 marks each.

Section - A

1. (a) Find the equations to the line through the point (1, 2, 3) parallel to the line $x - y + 2z = 5$, $3x + y + z = 6$.

   (b) A variable plane which remains at a constant distance 3p from the origin cuts the coordinate

axes in A, B, C. Show that the locus of the centroid of the triangle is
$x^{-2} + y^{-2} + z^{-2} = p^{-2}$. 3,2½

2. (a) Show that the equations of the perpendicular from the point (1, 6, 3) to the line $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3}$ are $\frac{x-1}{0} = \frac{y-6}{-3} = \frac{z-3}{2}$ and the foot of perpendicular is (1, 3, 5) and the length of perpendicular is $\sqrt{13}$.

(b) Find the equation of the planes through the points (2, 2, 1) and (9, 3, 6) and perpendicular to the plane $2x + 6y + 6z = 9$. 3, 2½

3. (a) Obtain in the symmetrical form the equations of the projection of the line $8x - y - 7z - 8 = 0 = x + y + z - 1$ on the plane $5x - 4y - z = 5$.

(b) Show that the shortest distance between any two opposite edges of the tetrahdron formed by the planes
$y + z = 0, z + x = 0, x + y = 0,$
$x + y + z = a$ is $\frac{2a}{\sqrt{6}}$
and that three lines of shortest distance intersect in the point $x = y = z = -a$. 3,2½

4. (a) Prove that the three planes
$2x + y + z = 3, x - y + 2z = 4, x + y = 2$
from a triangular prism and find the area of a normal section of the prism.

(b) Prove that the equaion
$2x^2 - 2y^2 + 4z^2 + 6zx + 2yx + 3xy = 0$
represents a pair of plane and find the angle between them. 3, 2½

Section - B

5. (a) A sphere of constant radius r passes through the origin O and cuts the axes in A, B, C. Prove that the locus of the foot of perpendicular from O to the plane ABC is given by

$$\left(x^2+y^2+z^2\right)^2\left(x^{-2}+y^{-2}+z^{-2}\right)^2 = 4r^2.$$

(b) Obtain the equations of the tangent planes to the sphere $x^2 + y^2 + z^2 + 2x - 4y + 6z - 7 = 0$ which intersect in the line
$6x - 3y - 23 = 0 = 3z + 2.$ 3,2½

6. (a) Find the equation of the radical plane of the spheres
$x^2 + y^2 + z^2 - 2x + 6z = 6$ and
$2(x^2 + y^2 + z^2) = 3y + 5.$
Hence show that the given sphere intersect in a circle.

(b) Find the equation of the sphere coaxial with the spheres
$2(x^2 + y^2 + z^2) + 3x - 4y + 5z - 7 = 0$, and
$x^2 + y^2 + z^2 + x + 2z = 0.$ 3,2½

7. (a) Find the equation to the cone whose vertex is the origin and base the circle $x = z, y^2 + z^2 = a^2$ and show that the section of the cone by the plane XOY is a hyperbola.

(b) Find the equation of the right circular cone generated when the stright line $2y + 3y = 6$ $x = 0$ revolves about z–axis. 3,2½

8. (b) Show that the angle between the lines $x + y + z = 0$, $ayz + bzx + cxy = 0$ is $\frac{\pi}{2}$, if $a + b + c = 0$.

(b) Find the equation of the cone with vertex (5, 4, 3) and the guiding curve $3x^2 + 2y^2 = 6$, $y + z = 0$. 3,2½

Section - C

9. (i) Find the equation of a sphere whose centre and radius are given.

(ii) Find the equation of the plane through $(x_1, y_1, z_1), (x_2, y_2, z_2), (x_3, y_3, z_3)$.

(iii) Find the equation of the sphere through the circle $x^2 + y^2 + z^2 = 9$, $2x + 3y + 4z = 5$ and the point (1, 2, 3).

(iv) Find the angle between the lines $\frac{x}{1} = \frac{-y}{0} = \frac{z}{-1}$ and $\frac{x}{3} = \frac{y}{4} = \frac{z}{5}$.

(v) Define projection of a line segment and projection of a plane area on a plane π.

(vi) Define Power of a point w.r.t. a sphere $x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$. Also write formula to find powder of the point (x_1, y_1, z_1).

(vii) Define Right circular cone with figure. 2×7=14