CO-ORDINATE GEOMETRY - III
Semester – I

Time Allowed : Three Hours] **[Maximum Marks : 40**

Note : The candidates are required to attempt *two* questions each from Section A and B carrying 7½ marks each and the entire Section C consisting of 10 short answer type questions carrying 10 makrs in all.

Section : A

1. Show that for $S = ax^2 + 2hxy + by^2 + 2gx + 2fy + c$. The trace t and discriminants Δ are invariants under any one of the transformations of the coordinates axes.
2. Transform the equatins $3x^2 + 2xy + 3y^2 + 18x + 22y + 50 = 0$ to the form $Ax^2 + By^2 = C$ by suitable transformation of axes.
3. Show that segement of the tangent to a parablola cut off between the directrix and the curve substends a right angle at the focus of the parabola.
4. Find the condition that the line $lx + my + n = 0$ may :
   (i) touch the parabola $x^2 = 4ay$. (ii) be normal to the parabola $x^2 = 4ay$.

Section : B

5. Find the asymptotes to the hyperbola $3x^2 - 5xy + 2y^2 + 5X + 11 - 8 = 0$. Also write donw the equation of its conjugate hyperbola.
6. If A and B are the extermities of the conjugate diameters on an ellipes $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, then show that the tangents at A and B meet on the ellipese $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$ and tha the locus of the middle point of AB is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{1}{2}$.
7. (i) Prove that the sum of the focal distances of any point on an ellipse is constant and equal to the major axis.
   (ii) Prove that the tangents at the extremities of a diameter of an ellipse are parallel to the conjugate diameter.
8. Prove that the locus of the middle points of nomal chords of the rectangular hyperbola $x^2 - y^2 = a^2$ is $(y^2 - x^2)^3 = 4a^2x^2y^2$.

Section : C

9. Do as directed :
   (i) Determine the polar coordinates of the point $\left(2, 2\sqrt{3}\right)$.
   (ii) Find the co-ordinates of the point (–2, 5) when axes are rotated through 30°, origin being unchanged.
   (iii) Find the equation of the normal to the parabola $y^2 = 12x$ which is perpendicular to the line $x - 3y + 6 = 0$.
   (iv) If e_1 and e_2 are eccentricities of a hyperbola and conjugate hyperbola, then prove that $\frac{1}{e_1^2} + \frac{1}{e_2^2} = 1$.
   (v) Determine k so that $y = x + k$ may touch the ellipse $2x^2 + 3y^2 = 1$.
   (vi) For what value of k the equation $\frac{x^2}{9-k} + \frac{y^2}{5-k} = 1$ represents an ellipse ?
   (vii) Find the ecentricity of the hyperbola $16(x-1)^2 - 9(y-2)^2 = 144$.
   (viii) Find the curve whose parametric equations are : $x = e^t + e^{-t}$, $y = e^t - e^{-t}$.
   (ix) Find equation of Director circle of the ellipse $16x^2 + 9y^2 = 144$.

(x) Find the pole of the line $y = 2x$ with respect to parabola $y^2 = 2x$.