

CALCULUS-II

(Re-Appeal April 2012)

Time : Three Hours

Maximum Marks : 100

Note : Attempt one question each from Section A, B, C and D carrying 20 marks each, and the entire Section E consisting of ten short answer type questions carrying 2 marks each.

Section-A

1. (a) Evaluate $\lim_{x \rightarrow a} \frac{x^n - a^n}{x - a}$

(b) What condition should be satisfied by the parameters a and b so that the function :

$$f(x) = \begin{cases} x - 1 & ; \text{for } x \leq 1 \\ ax^2 + bx & ; \text{for } x > 1 \end{cases}$$

is continuous at $x = 1$? Are a and b unique ?

10 + 10

2. (a) Find the nth derivative of $\frac{2x + 1}{(x + 1)^2 (x + 3)}$

(b) Find all the asymptotes of the following curve :
 $x^4 - y^4 + xy = 0$

Section-B

3. (a) Evaluate $\int x \sin^{-1} x \, dx$

(b) Obtain Reduction formula for $\int \cos^m x \sin nx \, dx$

10+10

4. (a) Prove that $\int_0^{\pi/2} \log \sin x \, dx = -\frac{\pi}{2} \log 2$

(b) Sketch the regions enclosed by the curves and find the area $y^2 = 4ax$, $x^2 = 4ay$. 10+10

Section-C

5. (a) Solve $x(1 + y^2)dx + y(1 + x^2)dy = 0$

(b) Solve $\frac{dy}{dx} + 2xy = x$ 10+10

6. (a) Show that $\frac{1}{xy}$ is an Integrating factor of the equation $y dx - x dy = 0$.

(b) Solve and examine for singular solution of the differential equation $y = p^2$. 10+10

Section-D

7. (a) Solve $(D^3 - 3D^2 + 4)y = e^{3x}$.

(b) Solve $(D^2 - 4D + 3)y = \sin 3x \cos 2x$ 10+10

8. (a) Solve $x \frac{d^2y}{dx^2} - \frac{dy}{dx} - 4x^3y = 8x^3 \sin x^2$ by the Change of Independent Variable method.

(b) Solve $\frac{d^2y}{dx^2} + 9y = \sin x$ by the method of Variation of Parameters. 10+10

Section-E

9. Do as directed :

(i) Evaluate $\lim_{x \rightarrow 2} \frac{2x^2 - 3x - 2}{x - 2}$

(ii) If $y = a \cos mx + b \sin mx$, show that $y^2 + m^2y = 0$.

(iii) What is oblique Asymptote ?

(iv) Evaluate $\int \sqrt{1 - 4x^2} dx$

(v) What is order and degree of a differential equation ?

(vi) Form the differential equation whose solution is $y = Ae^{2x} + Be^{-2x}$.

(vii) Solve $\frac{dy}{dx} + y = e^{-x}$

(viii) Solve $y = px + \sqrt{1 - p^2}$

(ix) Solve $(D^2 - 2D + 1)y = 0$

(x) Evaluate $\frac{1}{D}(x \sin x)$

10×2=20