|Maximum Marks: 40 Time Allowed: Three Hours Note: Attempt two questions from Section A and B carrying 8 marks each and the entire Section C consisting of 10 short answer type questions carrying 8 marks in all.

1.

Find all he asymptotes of the curve $x^3 + 2x^2y - xy^2 - 2y^3 + 4y^2 + 2xy + y - 1 = 0$ If $y = a \cos(\log x) + b \sin(\log x)$ show that $x^2y + (2n + 1)xy + (n^2 + 1)y = 0$ Determine a and b so that the curve $y = ax^3 + 3bx^2$ has a point of Inlexion at (-1, 2). Prove that the curve $y^2 = (x - a)^2(x - b)$ has at x = a, a node if a > b, a cusp if a = b and a conjugate point if a < b. 2

point if a < b. Trace the curve $y^2(a + x) = x^2(3a - a)$, a > 0. 3.

If the curve $y = a \log \sec \left(\frac{x}{a}\right)$, prove that the chord of curvature parallel to the axis of y is of consant 4. length.

Find the curvature at the point $\left(\frac{3a}{2}, \frac{3n}{2}\right)$ on the curve $x^3 + y^3 = 3axy$. Section: B

If $I_{m,n} = \int \sin^m x \cos^n x \, dx$, show that $I_{m,n} = \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \frac{n-1}{m+n} I_{m,n-2}$ 5.

Integrate $\int \frac{dx}{3\sin h x + 5\cos h x}$

Prove that the area common to the circles $r = a\sqrt{2}$ and $r = 2a\cos\theta$ is $a^2(\pi - 1)$. Find the length of the arc of the parabola $y^2 - 4y + 2x = 0$ which lies in the first quadrant.

(b)

If g is bounded and monotonic and tends to $0, x \to \infty$ and $\int f dx$ is bounded for $t \ge a$ then $\int f g dx$ 7. (a) is convergetn at ∞.

Examine the convergence of the improper integral $\int \frac{dx}{\sqrt{1-x^2}}$. (b)

Show that $\int_0^{\pi} \frac{\tan^{-1} ax}{x(1+x^2)} dx = \frac{\pi}{2} \log(1+a); a \ge 0.$ (a)

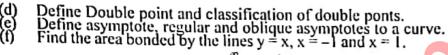
Prove that $\int_{0}^{\infty} x^{n} e^{-a^{2}x^{2}} dx = \frac{1}{2a^{n+1}} \left[\frac{n+1}{2} \right]$.

Do as directed: 9.

If $\sqrt{x} + \sqrt{y} = \sqrt{a}$, find the value of $\frac{d^2y}{dx^2}$ at x = a.

Define Concaviy or Convexity of a curve. Also what is point of inflexion?

(c) Write down the value of $\int \sin^8 0 d0$.



(d) Define Double point and classification of do Define asymptote, regular and oblique asymptote in the area bonded by the lines
$$y = x$$
, $x = -1$ (g) Examine the convergence of
$$\int_{c}^{\infty} \frac{dx}{x(\log x)^{3/2}}$$
.

(h) Show that the
$$\int_{0}^{a} e^{4x} dx$$
 converges.

(i) Prove that β (m, n) = β (n, m).

(j) Show that
$$\int_{0}^{\infty} \frac{x^{m-1} - x^{n-1}}{(1+x)^{m+n}} dx = 0$$
.

(h) Show that the
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$$\beta$$
 (m, n) = β (n, m).

(j) Show that
$$\int_{0}^{\infty} \frac{x^{m-1} - x^{n-1}}{(1+x)^{m+n}} dx = 0$$