CALCULUS–II

Semester–I

Time Allowed : Three Hours] **[Maximum Marks : 40**

Note : Attempt *two* questions from Section A and B carrying 8 marks each and the entire Section C consisting of 10 short answer type questions carrying 8 marks in all.

Section : A

1. (a) Find all he asymptotes of the curve $x^3 + 2x^2y - xy^2 - 2y^3 + 4y^2 + 2xy + y - 1 = 0$

   (b) If $y = a\cos(\log x) + b\sin(\log x)$ show that $x^2y_{n+2} + (2n+1)xy_{n+1} + (n^2+1)y_n = 0$

2. (a Detetrmine a and b so that the curve $y = ax^3 + 3bx^2$ has a point of Inlexion at $(-1, 2)$.

   (b) Prove that the curve $y^2 = (x-a)^2(x-b)$ has at $x = a$, a node if $a > b$, a cusp if $a = b$ and a conjugate point if $a < b$.

3. Trace the curve $y^2(a+x) = x^2(3a-a), a > 0$.

4. (a) If the curve $y = a\log\sec\left(\frac{x}{a}\right)$, prove that the chord of curvature parallel to the axis of y is of consant length.

   (b) Find the curvature at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$ on the curve $x^3 + y^3 = 3axy$.

Section : B

5. (a) If $I_{m,n} = \int \sin^m x \cos^n x\, dx$, show that $I_{m,n} = \frac{\sin^{m+1} x \cos^{n-1} x}{m+n} + \frac{n-1}{m+n} I_{m,n-2}$

   (b) Integrate $\int \frac{dx}{3\sinh x + 5\cosh x}$.

6. (a) Prove that the area common to the circles $r = a\sqrt{2}$ and $r = 2a\cos\theta$ is $a^2(\pi - 1)$.

   (b) Find the length of the arc of the parabola $y^2 - 4y + 2x = 0$ which lies in the first quadrant.

7. (a) If g is bounded and monotonic and tends to 0, $x \to \infty$ and $\int_a^t f\,dx$ is bounded for $t \geq a$ then $\int_a^\infty fg\,dx$ is convergetn at ∞.

   (b) Examine the convergence of the improper integral $\int_0^1 \frac{dx}{\sqrt{x - x^2}}$.

8. (a) Show that $\int_0^\infty \frac{\tan^{-1} ax}{x(1+x^2)} dx = \frac{\pi}{2}\log(1+a);\ a \geq 0$.

   (b) Prove that $\int_0^\infty x^n e^{-a^2x^2} dx = \frac{1}{2a^{n+1}}\Gamma\left[\frac{n+1}{2}\right]$.

Section : C

9. Do as directed :

   (a) If $\sqrt{x} + \sqrt{y} = \sqrt{a}$, find the value of $\frac{d^2y}{dx^2}$ at $x = a$.

   (b) Define Concaviy or Convexity of a curve. Also what is point of inflexion ?

   (c) Write down the value of $\int_0^{\pi/2} \sin^8\theta\, d\theta$.

(d) Define Double point and classification of double ponts.
(e) Define asymptote, regular and oblique asymptotes to a curve.
(f) Find the area bonded by the lines $y = x$, $x = -1$ and $x = 1$.

(g) Examine the convergence of $\int_{e}^{\infty} \frac{dx}{x(\log x)^{3/2}}$.

(h) Show that the $\int_{-\infty}^{a} e^{4x}\, dx$ converges.

(i) Prove that $\beta(m, n) = \beta(n, m)$.

(j) Show that $\int_{0}^{\infty} \frac{x^{m-1} - x^{n-1}}{(1+x)^{m+n}} dx = 0$.