DIFFERENTIAL EQUATIONS–I
Semester–I

Time Allowed : Three Hours] [Maximum Marks : 40

Note : The candidates are requied to attempt *two* questions from Section A and B carrying 8 marks each and the entire Section C consisting of 8 short answer type questions carrying 1 mark each.

Section : A

1. (i) Find an integrating factor for cos x cos y dx – 2 xin x sin y dy = 0 and then solve. 4

   (ii) Solve the differential equation : $\frac{dy}{dx} = (4x + y + 1)^2$ 4

2. Solve the differential equation : $\frac{d^2y}{dx^2} + \cot x \frac{dy}{dx} + 4y \operatorname{cosec}^2 x = 0.$ 8

3. (i) Using method of variation of parameters, solve : $\frac{d^2y}{dx^2} + 9y = \sec 3x.$ 4

   (ii) Define Cauchy's linear equation and hence solve it. 4

4. Solve the differential equation : $\frac{d^2y}{dx^2} + 16y = \sec 4x.$ 8

Section : B

5. (i) Using differential operator method, find the general solution of $\frac{dx}{dt} + \frac{dy}{dt} - x - 6y = e^{3t}, \frac{dx}{dt} + 2\frac{dy}{dt} - 2x - 6y = t.$ 4

   (ii) Prove that $F(l, m, p; 1) = \frac{\lceil(p)\lceil(p-l-m)}{\lceil(p-)\lceil(p-m)}$, where $p > l, p > m, p > l + m$ and $l, m > 0.$ 4

6. (i) For Bessel's function $J_n(x)$, prove that $J_{-\frac{1}{2}}(x) = \sqrt{\frac{2}{\pi x}} \cos x.$ 4

   (ii) Solve $\frac{d^2y}{dx^2} + \frac{1}{x}\frac{dy}{dx} + \left(144 - \frac{441}{x^2}\right)y = 0.$ 4

7. (i) Prove that $\int_{-1}^{1} [P_n(x)]^2 dx = \frac{2}{2n+1}$ if $m = n.$ 4

   (ii) Prove that $\int_0^{\pi/2} J_1(x \cos\phi) d\phi = \frac{1 - \cos x}{x}.$ 4

8. Solve in power series $\frac{d^2y}{dx^2} + (x-1)^2 \frac{dy}{dx} - 4(x-1)y = 0$ about $x = 1.$ 8

Section : C

9. Do as directed :

(i) Find the differential equation of all parabolas having vertex at origin and axis along positive y-axis.

(ii) Solve $(D^4 + 2D^3 + 3D^2 + 2D + 1)\, y = 0$.

(iii) Find particular integral of $(D^4 - 2D^3 + D^2)\, y = x^3$.

(iv) Define integrating factor of a differential equation.

(v) Define ordinary and singular points of a homogeneous linear differential equation.

(vi) Express $5x^2 - 3x + 6$ in terms of Legender's polynomials.

(vii) Show that $F\left(\frac{1}{2}, 1, \frac{3}{2}; x^2\right) = \frac{1}{2x} \log\left(\frac{1+x}{1-x}\right)$.

(viii) Verify that Legender Polynomial

$P_4(x) = \frac{35}{8}x^4 - \frac{30}{8}x^2 + \frac{3}{8}$ satisfies legendre's equation when n= 4. **8×1=8**