VECTOR ANALYSIS AND GEOMETRY-III

(Re-Appear April 2012)

Time : Three Hours Maximum Marks : 100

Note : Attempt one question each from Section A, B, C and D carrying 20 marks each, and the entire Section E consisting of ten short answer type questions carrying 2 marks each.

Section-A

1. (a) Transform the euqation $14x^2 - 4xy + 11y^2 - 36x + 48y + 41 = 0$ to rectangular axes inclined at an angle of $\tan^{-1}\left(\frac{1}{2}\right)$ to the original axes.

   (b) For what value of λ does the equation $3x^2 + 9xy + \lambda y^2 + 3x + 3y + 3 = 0$

represent two straight lines ? Find the angle between them.

(c) Find the equation of the chord of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ in terms of co-ordinates of its middle point (x_1, y_1) 7,7,6

2. (a) Reduce $36x^2 + 24xy + 29y^2 - 72x + 126y + 81 = 0$ to the standard form and classify the conic.
(b) Show that the lines joining the origin to the points of intersection of the line $x + 2y - 3 = 0$ and the circle $x^2 + y^2 - 2x - 2y = 0$ are at right angles to one another.
(c) Prove that the locus of the mid-points of focal chords of the parabola $y^2 = 4ax$ is $y^2 = 2a(x - a)$ 7,7,6

Section-B

3. (a) Show that the tangents at the extremities of a diameter of an ellipseare parallel of the conjugate diameter.
(b) Find the asymptotes of the hyperbola $x^2 + 24xy - 6y^2 + 28x + 36y + 16 = 0$ Also find the equation of the conjugate hyperbola.
(c) Prove that the locus of the mid-point of normal chords of the rectangular hyperbola $x^2 - y^2 = a^2$ is $(y^2 - x^2)^3 = 4a^2x^2y^2$ 6,7,7

4. (a) Show that the equation of a tangent line to the conic $\frac{l}{r}=1-e\cos(\theta-\gamma)$ at the point α is

$$\frac{l}{r}=1-e\cos(\theta-\gamma)+\cos(\theta-\alpha)$$

(b) Prove that the subnormal at any point of a parabola is constant and is equal to half the latus rectum.
(c) Find the condition that the line $\frac{l}{r}=A-\cos\theta+B\sin\theta$ may touch the circle $r = 2a\cos\theta$ 8,7,5

Section-C

5. (a) Show that the torque about the point A (3, –1, 3) of a force (4, 2, 1) through B (5, 2, 4) is (1, 2, –8)
(b) If $\vec{a}, \vec{b}, \vec{c}$ and $\vec{a}', \vec{b}', \vec{c}'$ are reciprocal system of vectors, prove that
(i) $\vec{a}\times\vec{a}'+\vec{b}\times\vec{b}'+\vec{c}\times\vec{c}'=0$
(ii) $\vec{a}.\vec{a}'+\vec{b}.\vec{b}'+\vec{c}.\vec{c}'=3$
(c) Find the equation of the sphere having its centre on the line $5y + 2z = 0 = 2x - 3y$ and passing through the points (0, –2, –4) and (2, –1, –1) 6, 7, 7

6. (a) Find the equation of the cylinder whose generators are parallel to the line $\frac{x}{1}=\frac{y}{2}=\frac{z}{3}$ and guiding curve is the ellipse $x^2 + 2y^2 = 1, z = 3$.
(b) Show that the general equation of a cone which touches the three co-ordinate planes is $\sqrt{fx}\pm\sqrt{gy}\pm\sqrt{hz}=0$ 10, 10

Section-D

7. (a) A tangent plane to the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$ meets the co-ordinate axes in points L, M and N. Prove that the centroid of the triangle LMN lies on $\frac{a^2}{x^2}+\frac{b^2}{y^2}+\frac{c^2}{z^2}=9$

(b) Find the equation to the generating lines of the hyperboloid $\frac{x^2}{4}+\frac{y^2}{9}-\frac{z^2}{16}=1$ which pass through the point (2, 3, –4) 10, 10

8. (a) Reduce the equation $2x^2 - 7y^2 + 2z^2 - 10yz + 6x + 12y - 6z + 5 = 0$ to standard form and identify the curve.

(b) If A1, A2 and A3 are the areas of three mutually perpendicular central sections of an ellipsoid, show that $\frac{1}{A_1^2}+\frac{1}{A_2^2}+\frac{1}{A_3^2}$ is a constant. 10, 10

Section-E

9. Do as directed :

(a) Transform to parallel axes through the point (1, –2) the equation $y^2 - 4x + 4y + 8 = 0$

(b) Find the projection of $\vec{a}=2\hat{i}-\hat{j}+\hat{k}$ on $\vec{b}=\hat{i}-2\hat{j}+\hat{k}$

(c) Find the condition that the line $lx + my + n = 0$ may be tangent to the parabola $y^2 = 4ax$

(d) Name the conic represented by $x=\frac{a(1-t^2)}{1+t^2}$, $y=\frac{2bt}{1+t^2}$

(e) Define Rectangular hyperbola and find its eccentricity.

(f) Find the co-ordinates of points in which the line $\frac{x+2}{4}=\frac{y+9}{3}=\frac{z-8}{-5}$ meets the sphere $x^2 + y^2 + z^2 = 49$

(g) Name the conicoids represented by the surfaces

(i) $\frac{x^2}{a^2}-\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ (ii) $\frac{x^2}{a^2}-\frac{y^2}{b^2}=\frac{2z}{c}$.

(h) Define Director circle of a conic. Write the equations of director circle in case of ellipse and hyperbola.

(i) Find the equation of the right circular cylinder whose axis is Z-axis and radius is a.

(j) Find the pole of the line $12x + 7y + 16 = 0$ w.r.t. the ellipse $4x^2 + 7y^2 = 8$ 2×10=20