

VECTOR ANALYSIS AND GEOMETRY – III

Time Allowed : Three Hours]

[Maximum Marks : 100

Note : The candidates are required to attempt *one* questions from Sections A, B, C and D carrying 20 marks each and the entire Section E consisting of 10 short answer type questions carrying 2 marks each.

Section : A

1. (a) Transform the equation $3x^2 + 2xy + 3y^2 + 18x + 22y + 50 = 0$ by shifting the origin to $(-2, -3)$ and then rotating the axes through 45° . 10
- (b) Show that the equation $x^2 + xy + y^2 + x - 4y + 1 = 0$ represents an ellipse. 10
2. (a) Reduce the equation $3x^2 - y^2 - z^2 + 6x + 6y - 2z - 2 = 0$ into an equation in which the first degree terms are absent. 12
- (b) Prove that the equation of the Chord of the circle $x^2 + y^2 = a^2$ in terms of its middle point (x_1, y_1) is given by $xx_1 + yy_1 = x_1^2 + y_1^2$. 8

Section : B

3. (a) Find the eccentricity of the hyperbola of which $2x - 3y = 0$ and $x = 2y$ is a pair of its conjugate diameters. 10
- (b) If P, D are extremities of conjugate diameters of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, find the locus of the points of intersection of tangents at P and D and middle point of PD. 10
4. (a) Find the asymptotes to the hyperbola : $3x^2 - 5xy - 2y^2 + 5x + 11y - 8 = 0$. 10
- (b) Find the directrix of the parabola $x^2 + 2xy + y^2 - 4x + 8y - 6 = 0$. 10

Section : C

5. (a) Find the equation of the sphere having the circle $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0$, $x + y + z - 3 = 0$ as a great circle. 10
- (b) If α is the semi-vertical angle of the right circular cone which passes through the lines $ox, oy, x = y = z$. Show that $\cos \alpha = (9 - 4\sqrt{3})^{-1/2}$. 10
6. (a) Find the equation of the enveloping cylinder of the sphere $x^2 + y^2 + z^2 - 2x + 4y = 1$, having its generators parallel to the line $x = y = z$. 14
- (b) Let $A = i + 2j - k, B = -i + j + k, C = i + k$, $D = -\left(\frac{\pi}{2}\right)i - \pi j + \left(\frac{\pi}{2}\right)k$. Which vectors, if any are (a) perpendicular, (b) parallel? Give reasons for your answers. 6

Section : E

9. Do as directed :
 - (i) If by a shift of origin to the point P, the point $(-4, 8)$ has new co-ordinates $(1, 15)$. Find the coordinates of P.
 - (ii) Find the equation of the polar of the point (x_1, y_1) with respect to the parabola $y^2 = 4ax$.
 - (iii) Prove that if tangents be drawn to the parabola $y^2 = 4ax$ from a point on the line $x + 4a = 0$, their chord of contact will subtend a right angle at the vertex.
 - (iv) If $|F| = 40 \text{ N}$, $|D| = 3 \text{ m}$, and $\theta = 60^\circ$, find the work done by F in acting from P to Q.
 - (v) Obtain the equation of a right circular cylinder whose radius is 4 and axis z-axis.
 - (vi) Find the Polar equation of a circle whose centre is $(1, \pi/4)$ and radius 3.

(vii) Find the equation of the tangent at the point α to the conic $\frac{r}{r} = 1 - e \cos \theta$ if equation of the Chord

Joining $P(\alpha)$ and $Q(\beta)$ on the conic is $\frac{r}{r} = 1 - e \cos \theta + \sec \frac{\alpha - \beta}{2} \cos \left(\theta - \frac{\alpha + \beta}{2} \right)$.

(viii) Find the equation of the pair of lines through the origin which represents the lines perpendicular to the pair of lines $ax^2 + 2hxy + by^2 = 0$.

(xi) Find the equation of tangent to the parabola $y^2 = 4ax$ at the vertex.

(x) Write down the name of surface represented by $2x^2 + y^2 + 3z^2 + 4x + 6z + 10 = 0$.

10×2=20