CALCULUS - I

Semester - 1

Time Allowed : 3 Hours]   [Maximum Marks : 36

Note :   The candidates are required to attempt *two* questions each from Section A and B carrying 5½ marks each and the entire Section C consisting of 7 short answer type questions carrying 2 marks each.

Section - A

1. What is an asymptote ? Find he horizontal as well as the vertical asymptotes of the curve $y = \dfrac{x+3}{x+2}$.   5½

2. What is a point o inflexion ? Show that the cubic curve $y = \dfrac{x^3}{6} - \dfrac{x^2}{2} + x$ has one point of inflexion and find hat point.   5½

3. Sketch the polar curve $r = 2a \cos 2\theta$.   5½

4. Define the circle of curvature and find he curvature of the helix.

$$r(t) = (a\cos t)\,\vec{i} + (a\sin t)\,\vec{j} + bt\,\vec{k}$$

$a, b, \geq 0$ and $a^2 + b^2 \neq 0$.

5½

Section - B

5. Evaluate $\int 2^{-x} \tanh 2^{1-x}\, dx$.

5½

6. Show that $\displaystyle\lim_{r \to \infty} \int_0^r \frac{dx}{x^2 + 4} = \frac{\pi}{8}$.

5½

7. Show that the series $\displaystyle\sum \frac{(-1)^{n-1}}{n \log_e^2 n}$ is absolutely convergent.

5½

8. For the Beta function B (u, v), prove that $B(u, v) = \dfrac{\Gamma(u)\Gamma(v)}{\Gamma(u+v)}$, where Γ is the Gamma function.

5½

Section - C

9. Attempt in short :
    (i)   What are the asymptotes of $y = \sec x$?
    (ii)   State the 2nd derivative test for concavity.
    (iii) Give an example of a graph which on any interval is concave up.
    (iv)   What is a carbdioid ? Draw a cardioid.

    (v)   Is $\displaystyle\int_1^\infty \frac{dx}{1+x^2}$ convergent ? Justify.

    (vi)   Use the substitution $2x = 7$ to evaluate $\displaystyle\int_0^\infty x^6 e^{-2x}\, dx$.

    (vii) State Abel's limit theorem for convergence of a power series.

7×2=14