CALCULUS-I

Semester-I

Time Allowed : 3 Hours] [Maximum Marks : 36

Note : The candidates are required to attempt *two* questions each from Section A and B carrying 5.5 marks each and the entire Section C consisting of 7 short answer type questions carrying 2 marks each.

Section - A

1. (a) . If f is defined by $f(x) = \begin{cases} 0 & \text{if } x \le 0 \\ x^3 & \text{if } x > 0 \end{cases}$. Prove that f' and f'' both exist for each x, where as f''(x) exist at each x except 0. 3

(b) If $f(x) = \tan x$, prove that

$$f^n(0) - n_c f^{n-2}(0) + n_c f^{n-4}(0) - - - - = \sin\frac{\pi}{2}.$$ 2.5

2. (a) Find position and nature of double point of $x^4 + y^3 + 2x + 3y^2 = .$ 3

(b) Show that the asymptotes of the cube curve $x^3 - xy^2 - 2xy + 2x - y - 1 = 0$ cut the curve in at most three points which lies on the line $3x - y - 1 = 0$ 2.5

3. (a) Trace the curve $y = -\frac{x^2}{1+x^2}$. 3

(b) If $\frac{1}{y^m} + y^{\frac{1}{m}} = 2x$, prove that $(x^2 - 1)(y_{n+2} + (2n + 1)\, x_{yn} + 1 + (n^2 - m^2)y_n = 0.$ 2.5

4. (a) Trace the curve $r = a + b\cos\theta$, $a > b$. 2.5

(b) Find radius of curvature at any point of the curve $x^{\frac{2}{3}} + y^{\frac{2}{3}} = a^{\frac{2}{3}}$. 3

Section - B

5. (a) If $T_n = \int_0^{\frac{\pi}{4}} \tan^n x\, dx, n > 1$, show that $T_n + T_{n-2} = \frac{1}{n-1}$. 3

(b) Find length of the curve $y = x^{\frac{2}{3}}$ from $x = -1$ to $x = 8$. 2.5

6. (a) Find area of the region outside the circle $r = 2a \cos\theta$ and inside the cardiod $r = a(1 + \cos\theta)$. 3

(b) Show that $\int_0^\infty \left(\frac{1}{x+1} - e^{-x}\right)\frac{dx}{x}$ is convergent. 2.5

7. (a) Prove that $\int_0^\infty \frac{x^{m-1}}{(a+bx)^{m+n}}dx = \frac{1}{a^n b^m}\beta(m, n)$, where a, b, m, n are all positive. 3

(b) If $a > 0$, $b > 0$ prove that $\int_0^\infty \frac{e^{-ax} - e^{-bx}}{x} = \log\frac{b}{a}$. 2.5

8. (a) Show that $\int_0^\infty \frac{y^2}{1+y^4}dy = \frac{\pi}{2\sqrt{2}}$. 3

(b) Prove that the improper integral $\int_a^b \frac{dx}{(x-a)^p}$ coverges, iff $p < 1$. 2.5

Section - C

9. (a) Find the derivative of $a^{2x} + \frac{x}{2x+1}$.

(b) Show that $y^2 = 8x$ has no asymptotes.

(c) Find radius of curvature of $s = a\tan\frac{\psi}{2}$.

(d) Find parallel asymtotes of $y = \frac{1}{(x-3)(x+2)}$.

(e) Show that (0, 0) is point of inflexion of $y = x^{1/3}$.

(f) Find value of $\Gamma\left(-\frac{1}{2}\right)$.

(g) Evaluate $\int\left(\frac{e^x + e^{-x}}{e^x - e^{-x}}\right)^2 dx$. $7\times2=14$